Organization and regulation of financial systems

S8 DMC

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Outline

- Introduction
  - Lessons from the crisis
  - What is a bank and what do banks do?
- Models for banking regulation
  - Deposit insurance
  - Lender of last resort: a simple model
- Capital reserves: the case of insurance
  - Optimal choice of capital reserve
  - Failure risk and insurance demand

Grading

- The evaluation of the course will be based on
  - an oral presentation
  - by groups of 3 to 4 students
  - on a theme linked to real-world regulation
- The list of themes
  - is available on Moodle
    (the allocation taking place there)
  - Two groups will work independently on each theme

Lessons from the (last) crisis

see *Tirole, in "Balancing the Banks"*, or

Beneplanc and Rochet: "Risk management in turbulent times"

▷ Last crisis

- ▶ since 1970: 112 banking crises, affecting 93 countries
- ▶ 51 international crises (affecting several countries)

▷ Financial madness?

- ▶ ECON 101: all economic agents
  (incl. managers and employees in financial industries)
- ▶ react to the information and incentives

▷ Bad incentives + bad information $\Rightarrow$ Bad behavior

What happened?

- Origin : home loans market
- then:
  - sale of assets at FIRE-SALE PRICES
  - unprecedented AVERSION TO RISK
  - FREEZING OF INTERBANK and bond market
- "government" REACTION: bail-out ("renflouement") of some of the largest banks and a major insurance company

An example: AIG

▷ Beginning of 2007

- $ 1 trillion of assets
- $ 110 billion revenue
- 74 million customers

▷ September 2008: emergency government assistance

- 2-year emergency loan of $ 85 billion
- gvt hold 79.9% of shares

⇒ 50% of U.S. GDP has been **GUARANTEED**, **LENT** or spent by the Fed, the US Treasury and other federal agencies

The role of subprime mortgages

- ▷ **Sub**prime mortgages ("prêt hypothécaires"): loans w/ difficulties in maintaining repayment schedule
  - ▶ higher interest rate
  - ▶ less favorable terms (collateral)

  to **COMPENSATE** for high risk

- ▷ losses on the US subprime market **SMALL** relative to previous figures ($1,000 billion, 4% of NYSE capitalization)

= detonator for a sequence of incentives and market **FAILURES** (asym. info. betw/ contracting parties) exacerbated by bad news

Other issues

- ▷ bad **REGULATION** $\rightarrow$ incentives to take risk
- ▷ **POLITICAL** resolution to favor real estate
  (to promote acquisition of homes by households)
- ▷ **MONETARY POLICY**: short term interest rate low
- ▷ excessive **LIQUIDITY**
  - ▶ international savings $\rightarrow$ US $\Rightarrow$ excess liquidity

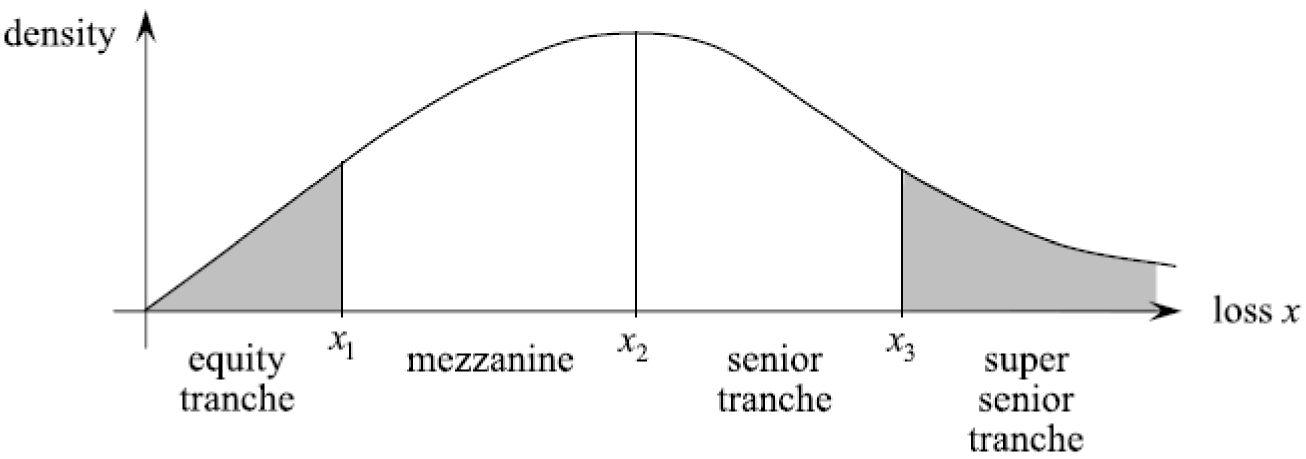
    $\Rightarrow$ **SECURIZATION** ("titrisation") to answer the demand

SECURIZATION

▷ Aim

- ▶ to refinance the lender → can finance other activities
- ▶ to fulfill the demand for securities
- ▶ to diversify and spread risk

▷ example: TRANCHING



[equity ("fond propre") tranche generally retained by the bank]

Securization: CDO

- Collateralized Debt Obligation
  - the bank issues bonds against investment
  - PRIORITIZED by different tranches
  - ex: 3 loans of nominal 1, each w/ proba 10% of default and 0 recovery in case of default
  - $\mathbb{P}(\text{i defaults}) = \binom{i}{n} p^i (1-p)^{n-i}$

    $\mathbb{P}(1\text{ d}) = 24.3\%$, $\mathbb{P}(2\text{ d}) = 2.7\%$, $\mathbb{P}(3\text{ d}) = 0.1\%$
  - equity tranche: loss up to 1
    mezzanine tranche: loss between $x_1 = 1$ and $x_2 = 2$
    senior: losses above $x_2 = 2$

SECURIZATION: CDS

- Credit Default Swap
  - contract between two parties
  - the **PROTECTION** buyers pays a period premium
  - to the protection seller who, in exchange,
  - commit to pay a fixed sum if a credit instrument (a bond or a loan) **DEFAULT**
- different for insurance
  - the buyer **DOESN'T NECESSARILY OWN** the credit instrument
  - the seller is **NOT A REGULATED** entity

SECURIZATION: ISSUES

▷ shift the RESPONSIBILITY away from the lender

⇒ less incentive to CONTROL

▷ asymmetry of INFORMATION

▷ laxity of credit-rating AGENCIES

▷ excessive maturity transformation

The Northern Rock example

- ▷ STRATEGY: invest in (apparently) safe tranches of Residential Mortgage Backed Securities (RMBS)
- ▷ financed by short term deposit
- ▷ PROBLEM: rumors (risk on RMBS) $\Rightarrow$ panic $\Rightarrow$ bank run

$\Rightarrow$ nationalization: injection of £23 billion

- ▷ lack of liquidity also led to default of Lehman Brothers (biggest default in the US history: \$ 613 bn of debt)

How to regulate?

- ▷ Basel accords: REQUIREMENT regarding the minimal level of CAPITAL or equity ("fonds propres")
- ▷ Basel I: requires 8% OF BANK CREDIT RISK
- ▷ Problems
  - ▶ OTHER RISKS? Liquidity risks? Off balance-sheet?
  - ▶ Risk MEASURE?
  - ▶ INFORMATION
  - ▶ INCENTIVES. Ex: managerial incentives (stock options). The CEO of Lehman Brothers earned $ 250 million between 2004 and 2007
  - ▶ SYSTEMIC institutions: Too Big To Fail

Basel II

▷ published in 2004, "implemented" in 2008

Pillar I Minimal Capital Requirement	Pillar II Supervisory Review Process	Pillar III Disclosure requirement
▷ Credit Risk ▷ Market Risk ▷ Operational Risk	▷ Regulatory framework ▶ Internal cap. adequacy ▶ Risk management ▷ Supervisory framework ▶ Evaluation of internal systems ▶ Assessment of risk profile	▷ Disclosure on capital, risk exposures, risk assessment process capital adequacy ▷ Comparability

▷ Basel III published in 2010, not yet fully implemented

▷ tries to also account for liquidity risk

▷ and SIFIs (Systemically important financial institutions)

What is a bank and what do banks do? (1)

see *Freixas and Rochet: "Microeconomics of banking"*

- ▷ Banking operations VARIED AND COMPLEX
- ▷ But a SIMPLE operational def (used by regulators) is

"a bank is an institution whose current operations consist in granting loans and receiving deposits from the public"

- ▷ CURRENT important: most firms occasionally lend money to customers or borrow from suppliers.
- ▷ BOTH LOANS AND DEPOSITS important: combination of lending and borrowing typical of commercial banks. Finance a significant share of loans through deposits $\rightarrow$ fragility.
- ▷ PUBLIC: not armed ($\neq$ professional investors) to assess safety financial institutions. Public good (access to safe and efficient payment system) provided by private institutions

What is a bank and what do banks do? (2)

- ▷ Protection of depositors + safety and efficiency of payment system → PUBLIC INTERVENTION
- ▷ Crucial role in ALLOCATION OF CAPITAL
  - ▶ efficient life-cycle allocation of household consumption
  - ▶ efficient allocation of capital to its most productive use
- ▷ before performed by banks alone; now fin. markets also
- ▷ 4 FUNCTIONS performed by banks
  - ▶ Offering liquidity and payment services
  - ▶ Transforming assets
  - ▶ Managing risks
  - ▶ Processing information and monitoring borrowers

Liquidity and Payment Services

- ▷ Without transaction costs (Arrow-Debreu): no need for money.
- ▷ FRICTIONS → more efficient to exchange goods for money.
- ▷ commodity money ("m. marchandise") → fiat money ("m. fiduciaire"): medium of exchange, intrinsically USELESS, guaranteed by some institution
- ▷ Role of BANKS
    - ▶ money change (exchange between different currencies issued by distinct institutions) ⇒ dvlp of trade
    - + management of deposits (less liquid, safer)
    - ▶ payment services: species inadequate for LARGE or at distance payments
    - → banks played an important part in clearing positions

Transforming Assets

Asset transformation can be seen from three viewpoints:

- ▷ convenience of **DENOMINATION** (size). Ex: small depositors facing large investors willing to borrow indivisible amounts.
- ▷ **QUALITY** transformation: better risk-return characteristics than direct investments (diversified portfolio, better info)
- ▷ **MATURITY** transformation: transforms short maturities (deposits) into long maturities (loans) $\rightarrow$ risk of illiquidity
  Solution: interbank lending and derivative financial instruments (swaps, futures)

Managing risk

▷ Credit risk ⇒ use of **COLLATERAL**

▷ Liquidity risk ⇒ interest rate

▷ Off-Balance-sheet risk: **COMPETITION** ⇒ more sophisticated contracts

- loan commitment, credit lines
- guarantees and swaps (CDS)
- hedging contracts ("opération de couverture")

▷ not real liability (or asset): **CONDITIONAL COMMITMENT**

⇒ need of careful **REGULATION**

Monitoring and information processing

▷ Problems resulting from **IMPERFECT INFORMATION** on borrowers.

⇒ Banks invest in technologies that allow them

- to **SCREEN** loan applicants and
- to **MONITOR** their projects

▷ Long-term relationships: mitigates **MORAL HAZARD**

A simple model with moral hazard

- ▷ Firms seek to finance investment projects of a size 1
- ▷ Risk-free rate of interest normalized to zero.
- ▷ Firms have **CHOICE** between
  - ▶ a good technology: G with proba. Π_G (0 otherwise)
  - ▶ a bad technology: B with proba. Π_B (0 otherwise)
- ▷ Only G proj. have positive net (expected) present value:

$$\Pi_G G > 1 > \Pi_B B$$

but $B > G$, (which implies $\Pi_G > \Pi_B$)

- ▷ Success verifiable, not choice of techno. (nor return)

$\rightarrow$ can promise to repay R (nominal debt) only if success

$+$ no other source of cash $\rightarrow$ repayment zero if fails

- ▷ value of R determines choice of **TECHNOLOGY**

In the absence of monitoring

▷ chooses G techno. iif gives higher expected profit:

$$\Pi_G(G-R) > \Pi_B(B-R)$$

▷ Since $\Pi_G > \Pi_B$ this is equivalent to

$$R < R_C \equiv \frac{\Pi_G G - \Pi_B B}{\Pi_G - \Pi_B}$$

⇒ Proba Π of REPAYMENT DEPENDS ON R:

$$\Pi(R) = \begin{cases} \Pi_G \text{ if } R \leq R_C \\ \Pi_B \text{ if } R > R_C \end{cases}$$

▷ Competitive equilibrium → $\Pi(R).R = 1$

▷ as $\Pi_B R < 1 \ \forall R < B$, ONLY POSSIBLE EQ.: G

▷ works only if: $\Pi_G R_C \geq 1$, i.e. R_C high enough

↔ if MORAL HAZARD NOT TOO IMPORTANT

▷ otherwise: no trade (no credit market)

Including monitoring

▷ at cost C, **BANKS** can prevent from using bad techno

⇒ new equilibrium interest rate: $\Pi_G R_m = 1 + C$

▷ bank lending appear at equilibrium if (as $R_m < G$):

- $\boxed{\Pi_G G > 1 + C}$ ↔ monitoring cost lower than the NPV
- $\boxed{\Pi_G R_C < 1}$ ↔ direct lending (less expensive) not possible

▷ that is for intermediate values of Π_G:

$$\Pi_G \in \left[\frac{1+C}{G}, \frac{1}{R_C}\right]$$

Conclusion

▷ Assuming the monitoring cost C small enough so that

$$\frac{1}{R_C} > \frac{1+C}{G}$$

▷ 3 possible regimes of the credit market at equilibrium:

- if $\Pi_G > \frac{1}{R_C}$: firms issue direct debt at rate $R_1 = \frac{1}{\Pi_G}$
- if $\Pi_G \in \left[\frac{1+C}{G}, \frac{1}{R_C}\right]$: borrow from **BANKS** at rate $R_2 = \frac{1+C}{\Pi_G}$
- if $\Pi_G < \frac{1+C}{G}$: credit market collapses (no trade eq.)

Possible extensions

▷ Dynamic model (2 dates) with **reputation**

- repayment at $t = 1 \rightarrow$ possibility of (cheaper) direct loan at $t = 2$
- $R^{t=1} < R_C$ (reputation $\downarrow$ moral hazard); $R_U^{t=2} > R_C$

▷ Use of **capital** (choice between capital and debt)

- well capitalized $\rightarrow$ direct loan
- intermediate capitalization $\rightarrow$ bank loan
- under-capitalized $\rightarrow$ no loan

$\rightarrow$ substituability between capital and monitoring

Recall: why to regulate? (1)

- ▷ In general: **Welfare** theorems
- ▷ Regulation iif **market failures**: externalities, asymmetric information
- ▷ Banks (or fin. intermediaries) solve some of these problems
- ▷ **BUT** create others:
  - ▶ liquidity risk: assets illiquid, liabilities liquid

<table>
<tr><th>Assets</th><th>Liabilities</th></tr>
<tr><td rowspan="2">Loans</td><td>Deposits</td></tr>
<tr><td rowspan="2">Capital (bonds)</td></tr>
<tr><td>Reserves</td></tr>
</table>

Recall: why to regulate? (2)

▷ to PROTECT CLIENTS (small depositors)

- ▶ $\neq$ other institutions: creditors = public

  $\rightarrow$ no monitoring power

- ▶ creditor of other firms: BANKS (can monitor)

+ CONFLICT OF INTERESTS btw/ manager and depositors managers take too much risk (not their mean of payment)

+ COST OF FAILURE: contagion + CONFIDENCE on the system of payment

$\Rightarrow$ Deposit Insurance + Lender of last resort + Capital ratio (+ Takeover ultimately)

Deposit Insurance

- to avoid bank panics and their social costs
- governments have established deposit insurance schemes: banks pay a premium to a deposit insurance fund
- ex Federal Deposit Insurance Corporation in the U.S.
    - created in 1933
    - in reaction to hundreds of failure in the 20s and 30s
- mostly public schemes
- pros
    - systemic risk $\rightarrow$ private sector not "credible"
    - take-off decisions = public
- cons: lack of competition
    - less incentive to extract info and price accurately

Deposit Insurance: a model

see *Freixas and Rochet: "Microeconomics of banking" (section 9.3)*

- 2 dates: $t = 0$ and $t = 1$
- at $t = 0$ the bank:
    - issues equity E
    - receives deposits D
    - loans L
    - pays deposit **INSURANCE PREMIUMS** P

Assets	Liabilities
Loans L	Deposits D
Insurance Premiums P	Equity E

▷ normalize the risk-free rate to 0

▷ at $t = 1$ the bank is liquidated

▷ depositors **COMPENSATED** if bank's assets insufficient

Assets	Liabilities
Loan repayments L_1	Deposits D
Insurance payments S	Liquidation value V

▷ from $t = 0$: V, S and L_1 are stochastic: $\widetilde{V}$, $\widetilde{S}$ and $\widetilde{L}_1$

▷ with $\widetilde{V} = \widetilde{L}_1 - D + \widetilde{S}$

▷ insurance pays difference betw/ deposits (to "pay back") and loan repayments:

$$\widetilde{S} = \max(0, D - \widetilde{L}_1)$$

▷ moreover from $t = 0$: $D = L + P - E$

- therefore:

$$\widetilde{V} = E + (\widetilde{L}_1 - L) + [\max(0, D - \widetilde{L}_1) - P]$$

- shareholders' value of the bank = its initial value + the increase in the value of loans + net subsidy (<0 or >0) received from deposit insurance.

- if **FOR EXAMPLE**

$$\widetilde{L}_1 = \begin{cases} X & \text{with prob. } \theta \\ 0 & \text{with prob. } 1-\theta \end{cases}$$

- the **EXPECTED GAIN** for the bank's shareholders is

$$\begin{aligned} \mathbb{E}(\Pi) &\equiv \mathbb{E}\left(\widetilde{V}\right) - E \\ &= \underbrace{(\theta X - L)}_{\text{net present value of loans}} + \underbrace{((1-\theta)D - P)}_{\text{net subsidy from insurance}} \end{aligned}$$

$$\mathbb{E}(\Pi) = (\theta X - L) + ((1-\theta)D - P)$$

- Problem: create moral hazard
  - Suppose P fixed, and
  - banks choose characteristics (θ, X) of projects
  - Then, within projects with same NPV: $\theta X - L = \text{cst}$
  - they choose those with lowest θ (i.e. HIGHEST RISK)
- Why?
  - P/D (premium rate) does not depend RISK TAKEN
  - as it was the case in the United States until 1991
  - then new system with RISK-RELATED premiums

Lender of Last Resort: a solution to coordination failure

see *Rochet and Vives, JEEA 2004*

Motivation

- ▷ Role of government (or IMF):
- ▷ lend to banks "ILLIQUID BUT SOLVENT"!
- ▷ redundant w/ interbank market?
  - ▶ Yes! If the market works well
  - ▶ i.e. without asymmetric information
  - ▶ if it can recognize solvent banks

Lender of Last Resort: the model

3 dates: $\tau = 0, 1, 2$

- ▷ at $\tau = 0$
  - ▶ bank possesses own funds E
  - ▶ collects uninsured deposits D_0 normalized to 1
    give $D > 1$ when withdrawn (independ. of the date)
  - ▶ used to finance investment I in risky assets (loans)
  - ▶ the rest is held in cash reserves K
- ▷ under normal circumstances: $I \to \widetilde{R}.I$ at $\tau = 2$
  deposits are reimbursed and shareholders get the difference
- ▷ BUT **ANTICIPATED WITHDRAWALS** (at $\tau = 1$) can occur
  depending on the signal received by depositors on $\widetilde{R}$
- ▷ if proportion $x > K$: bank has to **SELL** part of its assets

Assumptions

- Withdrawal decision taken by **FUND MANAGERS**
  - in general they prefer not to do so
  - BUT are penalized by the investors if the **BANK FAILS**
- consistent with observations
  - majority of deposits held by collective investment funds
  - remuneration of fund managers based on size not return
- Model: remun. based on whether take the "right decision"
  - if withdraw and not fail $\rightarrow -C$
  - if withdraw and fail $\rightarrow B$
- noting P the probability that bank fails: withdraw if

$$PB - (1-P)C > 0 \Leftrightarrow P > \gamma \equiv \frac{C}{B+C}$$

Signals and failure

At $\tau = 1$

▷ manager i **PRIVATELY** observes a signal $s_i = R + \varepsilon_i$ with ε_i i.i.d. and indep. from R

⇒ $x\%$ of the managers decide to withdraw

▷ if $x > K/D$ the bank has to **SELL** a volume y of its asset (repurchase agreement ∼ collateralized loan)

- if $y > I$: the bank **FAILS AT** $\tau = 1$
- if $R(I - y) < (1 - x)D$: the bank **FAILS AT** $\tau = 2$

Interbank market

- ▷ in case of **LIQUIDITY SHORTAGE** at $\tau = 1$
  - ▶ sell asset on repurchase agreement (or repo) market
  - ▶ informationally **EFFICIENT**: resale price depend on R
  - ▶ BUT cost (λ) of **FIRE-SALE** (or liquidity premium) the bank only gets a fraction $\frac{1}{1+\lambda}$ of its asset value

$$\Rightarrow y \quad / \quad \frac{Ry}{1+\lambda} = [xD - K]_+$$

$$\Leftrightarrow y = (1+\lambda)\frac{[xD-K]_+}{R}$$

- ▷ λ is key to this analysis
- ▷ reflects e.g. moral hazard: 2 reasons for selling asset
  - ▶ needs liquidity or wants to get rid of bad loans (value 0)
  - ▶ $\frac{1}{1+\lambda}$ is then the proba of the former

AIM OF THE MODEL

▷ want to show that interbank market DOES NOT SUFFICE

▷ to prevent EARLY CLOSURE of the bank

▷ and so that we need a LENDER OF LAST RESORT

▷ if R small (close to insolvency) or λ large (liquidity shortage)

▷ even with interbank market: early closure at $\tau = 1$

▷ Now: early closure $\rightarrow$ physical LIQUIDATION of assets

$\Rightarrow$ cost of liquidation ($\neq \lambda$)

▷ model: if a bank closes at $\tau = 1$, liquidation value νR with $\nu << \frac{1}{1+\lambda}$

Bank runs and solvency (1)

▷ if $xD \leq K$: no sale of assets at $\tau = 1$

$\Rightarrow$ failure at $\tau = 2$ iif $RI + K < D \Leftrightarrow R < \frac{D-K}{I} \equiv R_S$

▷ if $K < xD \leq K + \frac{RI}{1+\lambda}$: partial sale of assets at $\tau = 1$

$\Rightarrow$ failure at $\tau = 2$ iif

$$RI - (1+\lambda)(xD-K) < (1-x)D \Leftrightarrow R < R_S + \lambda \frac{xD-K}{I} \equiv R_F(x)$$

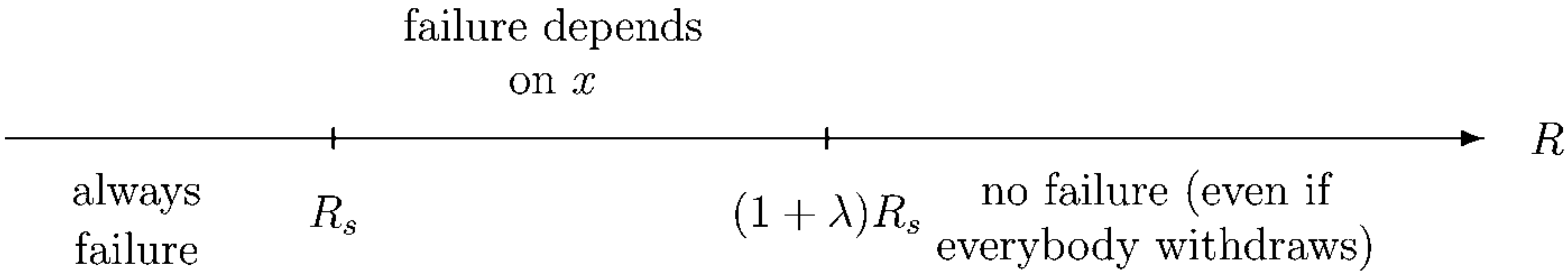
$\rightarrow$ Because of λ, **SOLVENT** banks ($R > R_S$) can fail

if $R > (1+\lambda)R_S$, never fails (even $x = 1$): super solvent

▷ if $xD > K + \frac{RI}{1+\lambda}$: failure at $\tau = 1$

$\Leftrightarrow R < (1+\lambda)\frac{xD-K}{I} \equiv R_{EC}(x)$

Bank runs and solvency (2)

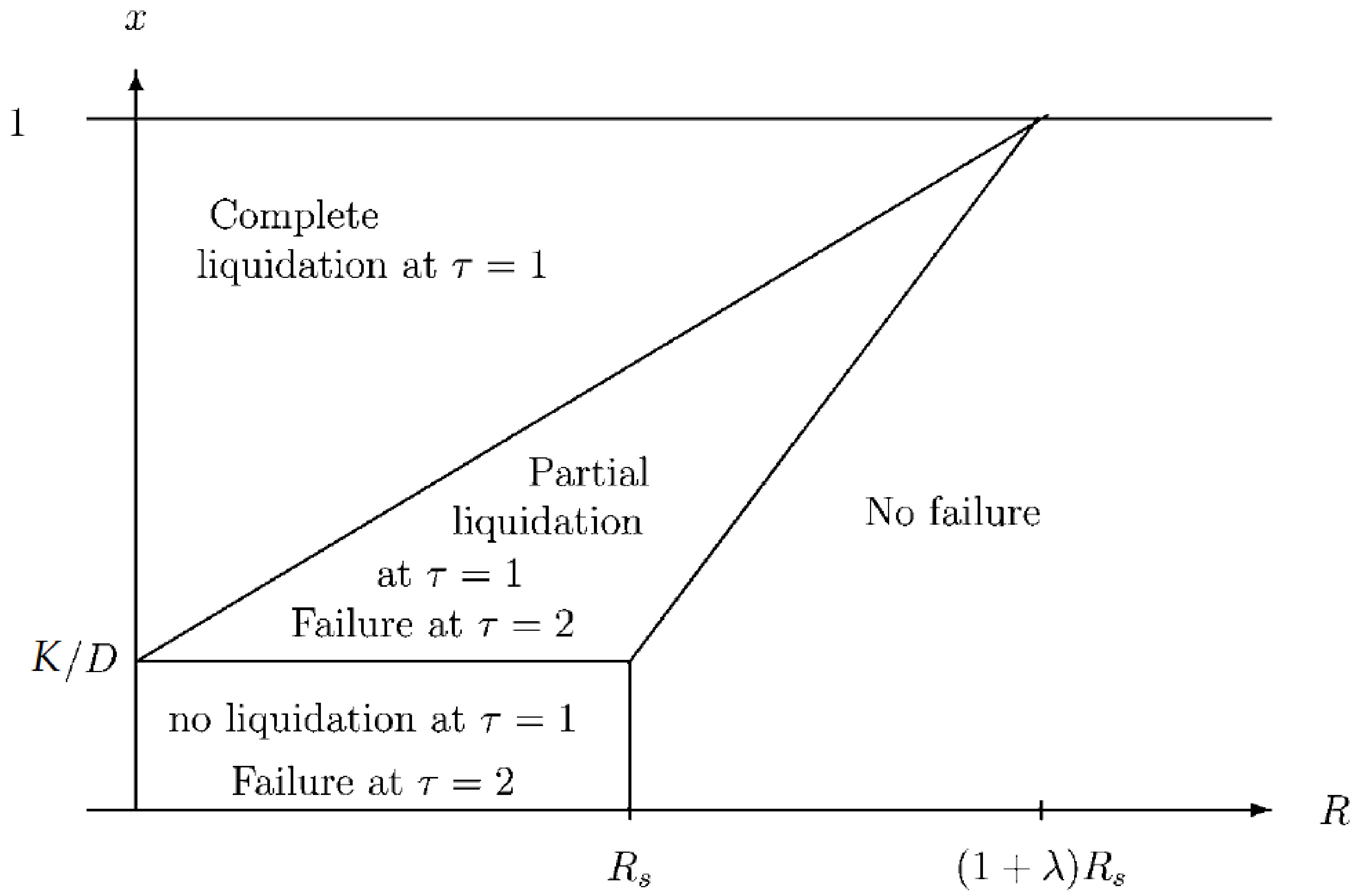


- using the **LIQUIDITY RATIO**: $k = K/D$, we have:

$$R_S = \frac{1-k}{I}D,\ R_F(x) = R_S\left(1+\lambda\frac{[x-k]_+}{1-k}\right),\ R_{EC}(x) = R_S\,(1+\lambda)\,\frac{[x-k]_+}{1-k}$$

- $x \leq 1 \Rightarrow R_F(x) > R_{EC}(x)$
- **EARLY CLOSURE IMPLIES FAILURE**
  (the converse is not true)

Bank runs and solvency (3)

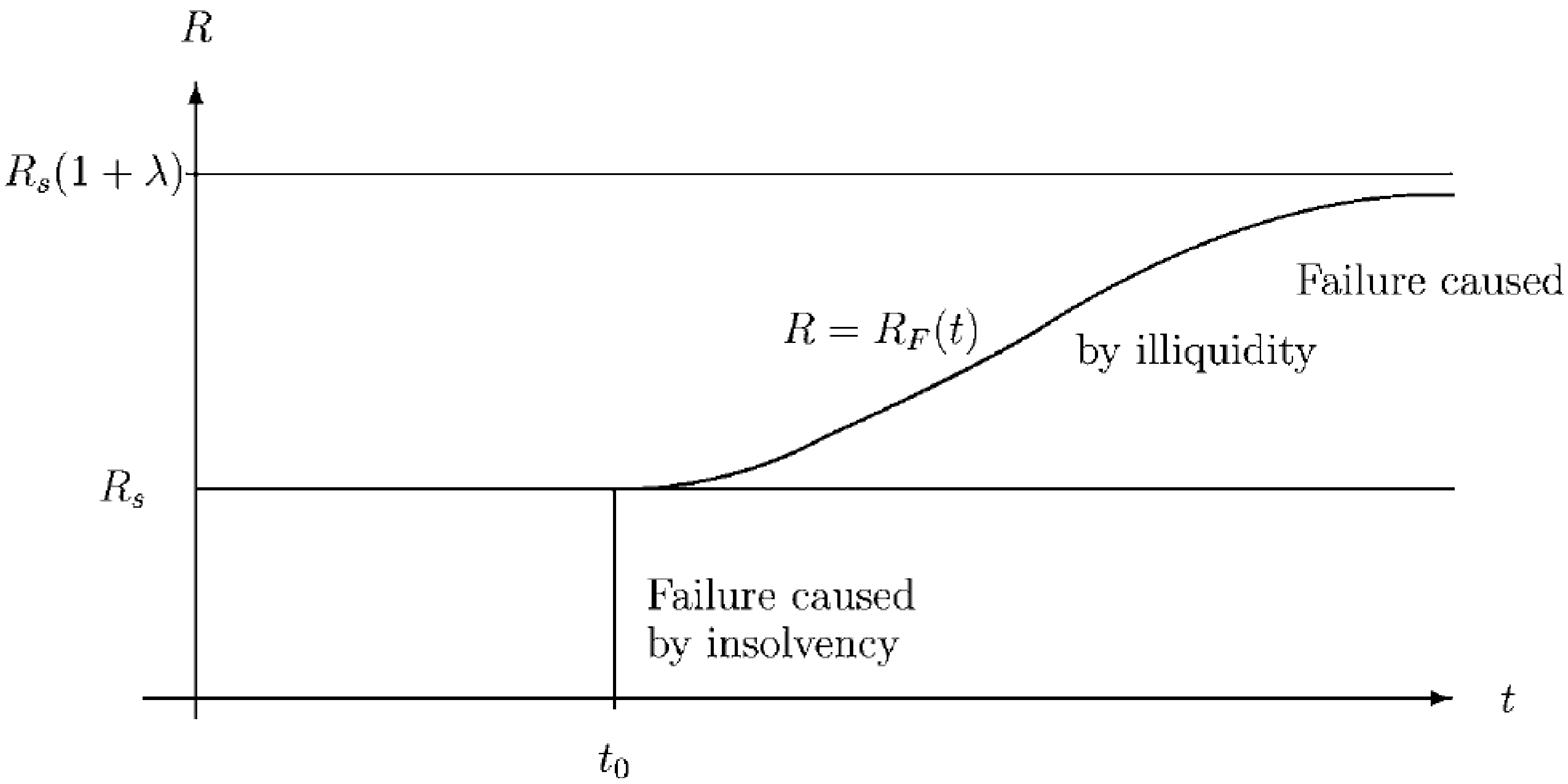


Equilibrium of the Investors' Game

- ▷ How is x determined?
- ▷ without loss of generality, assume a threshold strategy for all managers
- ▷ withdraw if **SIGNAL** $s < t$
- ▷ i.e. with proba: $\mathbb{P}(R + \varepsilon < t) = F(t - R)$ where F is the c.d.f of ε
- ▷ this proba. also equals the proportion of withdrawals $x(R, t)$
- ▷ moreover, we assumed that managers withdraw if
- ▷ the **PROBABILITY OF FAILURE**: $P(s, t) > \gamma$

$\Leftrightarrow \mathbb{P}\Big(R < R_F(x(R,t)) \mid s\Big) > \gamma \Leftrightarrow G(R_F(t) \mid s) \geq \gamma$
where $G(. \mid s)$ is the c.d.f. of R conditional on signal s

▷ now, as $x(R,t) = F(t-R)$, R_F is implicitly defined by:
$$R = R_S\left(1 + \lambda\left[\frac{F(t-R)-k}{1-k}\right]_+\right)$$



▷ with $t_0 / R_F = R_S$, i.e. $t_0 = R_S + F^{-1}(k)$
if $t \geq t_0$, "too many" failures $\rightarrow$ need for a LOLR

Strategic complementarity

▷ natural to assume $G(r \mid s)$ decreasing in s:
the higher s, the lower the proba that $R < r$

$\Rightarrow P(s,t)$ decreasing in s, increasing in t
$\big(P(s,t)$: proba. of failure when signal s and threshold $t\big)$

$\Rightarrow P(s,t) > \gamma \Leftrightarrow s < \overline{s}$ with $\overline{s}/P(\overline{s},t) = \gamma$, i.e. $\overline{s} = S(t)$
with $S'(t) = -\dfrac{\partial P/\partial t}{\partial P/\partial s} \geq 0$

$\Rightarrow$ a higher threshold t BY OTHERS induces a manager to use a HIGHER THRESHOLD also

Bayesian equilibrium (1)

- we look for a **STRATEGY** such that the equilibrium is consistent with the **BELIEFS**
- Managers withdraw if $P(s, t) > \gamma$ and withdraw if $s < t$
- **CONSISTENT** iif $t^*/P(t^*, t^*) = \gamma$
- then, as $P(s, t)$ decreasing in s:
  - $s < t^* \Rightarrow P(s, t^*) > \gamma \Rightarrow$ withdraw
  - $s > t^* \Rightarrow P(s, t^*) < \gamma \Rightarrow$ not withdraw

Bayesian equilibrium (2)

▷ The equilibrium (R_F^*, t^*), where

- ▶ t^* is the equilibrium **WITHDRAWAL THRESHOLD**
- ▶ R_F^* is the equilibrium **RETURN THRESHOLD**

is therefore determined by:

$$\begin{cases} G(R_F^* \mid t^*) & = \gamma \\ \quad R_F^* & = R_S \left(1 + \lambda \left[\frac{F(t^* - R_F^*) - k}{1 - k}\right]_+\right) \end{cases}$$

▷ 1st eq: if $s = t^*$, $\mathbb{P}(R < R_F^* \mid s) = \gamma$ (def of t^*)

▷ 2nd eq.: given t^*, R_F^* is the return threshold, below which failure occurs (def of R_F^*)

Gaussian case

- $\triangleright$ to go further, we assume
- $\triangleright$ $R \sim \mathcal{N}\left(\overline{R}, 1/\alpha\right)$
- $\triangleright$ $\varepsilon \sim \mathcal{N}\left(0, 1/\beta\right) \Rightarrow F(x) = \Phi(\sqrt{\beta}x)$
- $\triangleright$ we look for $G(R \mid s) = G(R \mid R + \varepsilon)$. As
  - ▶ $R + \varepsilon \sim \mathcal{N}\left(\overline{R}, 1/\alpha + 1/\beta\right)$, and
  - ▶ $\mathrm{cov}(R, R + \varepsilon) = \mathrm{Var}(R) = 1/\alpha$
- $\triangleright$ we have $R \mid R + \varepsilon \sim \mathcal{N}\left(\frac{\alpha\overline{R}+\beta s}{\alpha+\beta}, \frac{1}{\alpha+\beta}\right)$
- $\triangleright$ that is $G\left(R_F^* \mid t^*\right) = \Phi\left(\sqrt{\alpha+\beta}R_F^* - \frac{\alpha\overline{R}+\beta t^*}{\sqrt{\alpha+\beta}}\right)$

The equilibrium

▷ The equilibrium is then characterized

▷ by a pair (t^*, R_F^*) such that

$$\begin{cases} \Phi\left(\sqrt{\alpha+\beta}R_F^* - \frac{\alpha\overline{R}+\beta t^*}{\sqrt{\alpha+\beta}}\right) = \gamma \\ R_F^* = R_S\left(1+\lambda\frac{\Phi(\sqrt{\beta}(t^*-R_F^*))-k}{1-k}\right) \end{cases}$$

▷ and we can prove (proof omitted) that

Proposition. When β (precision of private signal) large enough relative to α (prior precision):

$$\beta \geq \frac{1}{2\pi}\left(\frac{\lambda\alpha D}{I}\right)^2 \equiv \beta_0$$

unique t^* such that $P(t^*, t^*) = \gamma$. The investor's game then has a unique equilibrium: a strategy with threshold t^*.

Coordination failure

- Failure caused by illiquidity (coordination failure) if $t^* > t_0$
- with t^* such that: $\Phi\left(\sqrt{\alpha+\beta}R_F^* - \frac{\alpha\overline{R}+\beta t^*}{\sqrt{\alpha+\beta}}\right) = \gamma$
- if $t^* \leq t_0$: **NO COORDINATION FAILURE**, i.e. $R_F^* = R_S$.
  In this case:
  $$t^* = \frac{1}{\beta}\left((\alpha+\beta)R_S - \sqrt{\alpha+\beta}\phi^{-1}(\gamma) - \alpha\overline{R}\right)$$
- as $t_0 = R_S + \frac{1}{\sqrt{\beta}}\phi^{-1}(k)$
- an equilibrium with $t^* \leq t_0$ occurs iif:
  $$(\alpha+\beta)R_S \leq \sqrt{\alpha+\beta}\phi^{-1}(\gamma) + \alpha\overline{R} + \beta R_S + \sqrt{\beta}\phi^{-1}(k)$$

Liquidity ratio and coordination failure

▷ That is iif:

$$k \geq \Phi\left(\frac{\alpha}{\sqrt{\beta}}\left(R_S - \overline{R}\right) - \sqrt{1 + \frac{\alpha}{\beta}\Phi^{-1}(\gamma)}\right) \equiv \overline{k}$$

Proposition. There is a critical liquidity ratio $\overline{k}$ of the bank such that, for $k = \frac{K}{D} \geq \overline{k}$ **only insolvent banks fail** (there is no coordination failure).

▷ if $k < \overline{k}$ solvent but illiquid banks fail

Probability of failure

▷ In this last case R_F^* is defined by:

$$\begin{cases} \Phi\left(\sqrt{\alpha+\beta}R_F^* - \frac{\alpha\overline{R}+\beta t^*}{\sqrt{\alpha+\beta}}\right) = \gamma \\ R_F^* = R_S\left(1+\lambda\frac{\Phi(\sqrt{\beta}(t^*-R_F^*))-k}{1-k}\right) \end{cases}$$

$$\Leftrightarrow \begin{cases} -\sqrt{\alpha+\beta}\Phi^{-1}(\gamma)+(\alpha+\beta)R_F^*-\alpha\overline{R}-\beta t^* = 0 \\ t^* = R_F^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}\left(\frac{1-k}{\lambda R_S}\left(R_F^*-R_S\right)+k\right) \end{cases}$$

$$\Leftrightarrow \alpha\left(R_F^*-\overline{R}\right) - \beta\Phi^{-1}\left(\frac{1-k}{\lambda R_S}\left(R_F^*-R_S\right)+k\right) - \sqrt{\alpha+\beta}\Phi^{-1}(\gamma) = 0$$

▷ As the l.h.s is decreasing in R_F^* for $\beta \geq \beta_0$ we have

Proposition. R_F^* – and therefore the proba of FAILURE – is decreasing in the liquidity ratio k, the critical withdrawal probability γ, and of the expected return $\overline{R}$ and increasing in the fire-sale premium λ and the face value of debt D.

How to avoid failure caused by illiquidity?

- ▷ theoretical possibility of a solvent bank being illiquid as a result of coordination failure on the interbank market.

- ▷ 2 possibilities (for a central bank or a gvt) to eliminate that:
  - ▶ lower bond on the liquidity ratio k: $\overline{k}$
  - ▶ decrease λ through:

    LIQUIDITY INJECTION (as for ex after Sept 11)

    DISCOUNT-RATE lending (ex. Fed '08, low rate but stigma)

Discount-rate lending (1)

▷ fixing $k \geq \overline{k}$: costly in terms of "returns":
$I + K = 1 + E \Rightarrow$ high K means **LOW INVESTMENT**

▷ what to do if $k < \overline{k}$?

- assume that the central bank lends at rate $r \in (0, \lambda)$ without limit, BUT only to **SOLVENT** banks
- Central bank not supposed to subsidize: $r > 0$
- and assumed to perfectly observe $R \leftarrow$ **SUPERVISION**

$\Rightarrow$ Optimal strat. for a bank = lend exactly $D(x-k)_+$

$\rightarrow$ failure in $\tau = 2$ iif

$$RI < (1-x)D + (1+r)(x-k)D$$

Discount-rate lending (2)

▷ That is, as $R_S = \frac{D-K}{I} = \frac{D(1-k)}{I}$, iif

$$R < R_S\left(1 + r\frac{[x-k]_+}{1-k}\right) \equiv R^*$$

(same as R_F^* with r instead of λ)

⇒ fully **EFFICIENT** ($R^* = R_S$) if r arbitrarily close to 0

+ central bank **LOSES NO MONEY** (loan repaid at $\tau = 2$) as only lends to solvent banks ($R > R_S$)

⇒ possible!

Possible extensions

- including moral hazard
  - investment in risky assets requires supervision
  - supervision effort by bank manager $e = \{0, 1\}$, $e = 1$ costly
  - $e = 0 \Rightarrow R \sim \mathcal{N}\left(\overline{R_0}, \frac{1}{\alpha}\right)$; $\quad e = 1 \Rightarrow R \sim \mathcal{N}\left(\overline{R}, \frac{1}{\alpha}\right)$
    with $\overline{R} > \overline{R_0}$
  - Result: the use of **SHORT-TERM** debt is optimal
    allowing withdraw at $\tau = 1$ discipline bank managers

- **ENDOGENIZING** $k = K/D$ (reserves chosen by the bank)

INSURANCE, FAILURE AND RESERVES

see *Rees, Gravelle and Wambach, The Microeconomics of Insurance, section 3.2*

- ▷ insurance = **PROMISE** (against a premium) to pay coverage in case of accident
- ▷ how to make sure this promise is kept? i.e. the insurance has enough reserve to pay coverage?
- ▷ has to ensure insurance doesn't fail
- ▷ as banks: **CREDITORS** of insurance companies are policy-holders

→ **CANNOT MONITOR** their insurance company

⇒ Existence of solvency **RULES** and **REGULATION** authorities

The model

- an insurer offers a contract to n **IDENTICAL** individuals
  same risk (distribution of claims identical), same preferences
- assume: **INDEPENDENT** risks ($\rightarrow$ i.i.d.)
  not necessary to determine aggregate loss but simplifies
- $\widetilde{C}_i$ distrib of ind claims i.i.d.: mean μ and variance σ^2
  $\Rightarrow \widetilde{C}^n = \sum_{i=1}^{n} \widetilde{C}_i$ distrib of aggregate claims, random var of mean $n\mu$

$\Rightarrow$ if premium sets to μ ("fair" premium) on each contract and insurance costs are zero
it will just **BREAK EVEN** ("rentable") in expected value:

$$\mathbb{E}(\text{Profit}) = \mathbb{E}\left(n\mu - \widetilde{C}^n\right) = n\mu - \mathbb{E}\left(\widetilde{C}^n\right) = 0$$

The need of reserves

▷ However

$$\begin{aligned}\mathrm{Var(Profit)} &= \mathrm{Var}\left(\widetilde{C}^n\right) = \mathbb{E}\left(\left(\sum_{i=1}^n \widetilde{C}_i - n\mu\right)^2\right) \\ &= \mathbb{E}\left[\left\{\sum_{i=1}^n \left(\widetilde{C}_i - \mu\right)\right\}^2\right] = \sum_{i=1}^n \mathbb{E}\left[\left(\widetilde{C}_i - \mu\right)^2\right] \\ &= n\sigma^2\end{aligned}$$

is positive and linearly **INCREASING IN n**

▷ no convergence: $\forall n$, we can have $\widetilde{C}_n <> n.\mu$

$\Rightarrow$ to **AVOID INSOLVENCY**

(when claims costs exceed funds available to meet them)

insurance have to carry **RESERVES**.

RUIN PROBABILITY (1)

▷ reasonable to assume maximum cover $C_{\max}$ per contract

⇒ maximum possible aggregate claims cost: $nC_{\max}$

⇒ if premium P and reserves $K_{\max} = n(C_{\max} - P)$:
ZERO PROBABILITY OF INSOLVENCY

▷ However, IN PRACTICE:

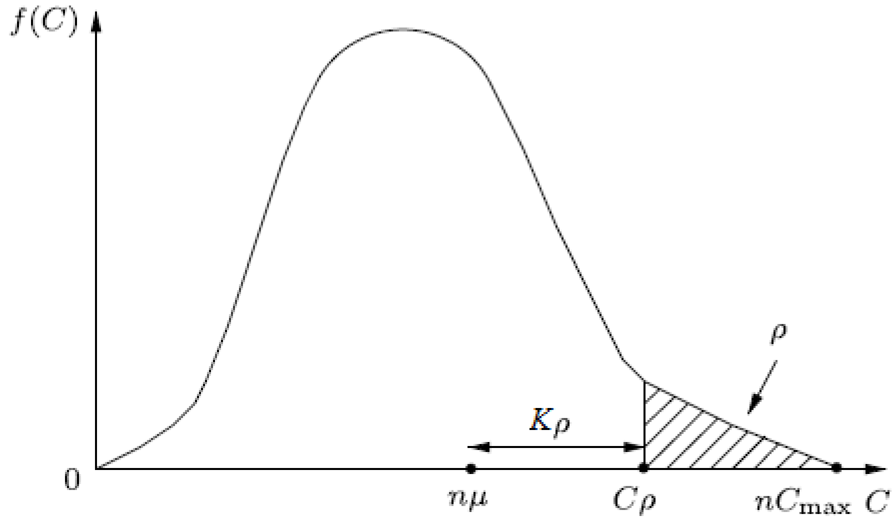
- proba. total claims near $nC_{\max}$ extremely small
- raising capital of $K_{\max}$ extremely costly

⇒ insurers choose a so-called RUIN PROBABILITY ρ
and given the distribution of $\widetilde{C}^n$ choose a level of reserves:
$K(\rho) = C_\rho - nP$ with $C_\rho \; / \; \mathbb{P}\left(\widetilde{C}^n > C_\rho\right) = \rho$

Ruin probability (2)

▷ reserves / proba. ρ to be insolvent

▷ that is, when $P = \mu$ (fair premium)



How is ρ determined?

- Trade-off between
  - the costs associated with the RISK OF INSOLVENCY depends on buyers' PERCEPTIONS of this risk
  - and the cost of holding reserves
- explored in more detail in the NEXT SECTIONS

The implications of the Law of Large Numbers

▷ let $C_1, C_2, ..., C_n$ the realizations of claims for n ind. (random sample from a distrib with mean μ and var σ^2)

▷ let $\overline{C_n} = \frac{1}{n}\sum_{i=1}^{n} C_i$ be the sample mean or the average LOSS PER CONTRACT

▷ Law of Large Numbers → $\forall\varepsilon > 0, \lim_{n\to\infty} \mathbb{P}\left(|\, \overline{C_n} - \mu \,| < \varepsilon\right) = 1$: for sufficiently large n, virtually certain that the LOSS PER CONTRACT equals μ, mean of individual loss distribution

▷ Moreover, $\text{Var}\left(\overline{C_n}\right) = \mathbb{E}\left(\left(\frac{1}{n}\sum \widetilde{C}_i - \mu\right)^2\right) = \frac{1}{n}\sum_{i=1}^{n} \sigma^2 = \frac{\sigma^2}{n}$

⇒ the variance of realized loss per contract goes to 0 as $n \to \infty$

INTERPRETATION

- ▷ as the number of CONTRACTS SOLD becomes very large,
- ▷ risk that realized loss per contract exceeds fair premium becomes vanishingly small.

≈ ECONOMY OF SCALE

- ▶ although variance of aggregate claims increases with n $\rightarrow$ the reserves have to increase in absolute amount)
- ▶ the required reserve per contract tends toward zero

- ▷ required reserves increase LESS THAN PROPORTIONATELY with size of the insurer (number of contracts)

Exercise

Consider a portfolio of 400,000 identical contracts for which

- ▷ the number of accidents per contract (N_i) can be approximated by a $\mathcal{P}(0,07)$
- ▷ the expected value of claims per accident $\mathbb{E}(C_{ij}) = 14500$ €
- ▷ with a standard error $\sigma(C_{ij}) = 130000$ €
- ▷ N_i are assumed to be i.i.d ; C_{ij} are assumed to be indep. from N_i, $\forall j$; given N_i, C_{ij} are assumed to be i.i.d $\forall i, j$
  - ▶ Calculate the fair premium of a contract
  - ▶ Calculate the standard error of the annual claims on a contact
  - ▶ Calculate the amount of reserves that makes the ruin probability lower than 5% (assuming fair premia)

Optimal choice of reserves

see *Rees and Wambach, The Microeconomics of Insurance, section 3.5*

- regulation: protect policyholders against risk of failure
- REVERSE PRODUCTION CYCLE $\rightarrow$ risk of fraud: once premiums paid insurer can RUN OFF
- But large, well-established companies
  - that wish to remain in business for the LONG TERM
  - would not need detailed regulatory intervention,
  - to ensure they carry enough reserves to meet obligations
- want to model these effects

The key assumptions

1. **Limited liability** ("engagement limité")
   - a shareholder is **liable** for the debts of a company **only** up to the value of his shareholding

$\Rightarrow$ unregulated insurer may find optimal to put **no reserves** and fail as soon as claims exceed collected premiums (more so if reserves are costly)

2. **Increasing failure rate**

$$\frac{d}{dC}\frac{f(C)}{1-F(C)} > 0$$

met by virtually all insurance loss-claims distributions

THE MODEL (1)

- ▷ consider an insurance company in business for the long term
- ▷ so taking decisions over an INFINITE TIME HORIZON
- ▷ with a sequence of DISCRETE TIME PERIODS (say years).
- ▷ At the beginning of EACH YEAR
- ▷ decide on a level of reserve capital K
- ▷ given the distribution of claims C: $F(C)$
  with (differentiable) density $f(C)$, defined over $[0, C_{max}]$
- ▷ COSTLESS reserves: owns (enough) capital but has to decide whether to invest or to commit it in the insurance business

THE MODEL (2)

▷ premium income P **EXOGENOUS** (independent of K)
buyers do not perceive relationship reserves and insolvency and act **AS IF NO SOLVENCY RISK**

▷ P collected at the **BEGINNING** of the period
and invested with K in **RISKLESS** asset (return $r > 1$)

⇒ At the end of the period assets: $A = (P + K)r$

- if $A > C$: **REMAINS IN BUSINESS**
  and receives continuation value V (expected present value of returns from insurance business over all future periods)
- if $A < C$: **DEFAULTS**
  A used to pay claims, loses V
  limited liability: doesn't pay claims above A

Optimal reserves (1)

- $C < C_{max} \Rightarrow$ can always choose to guarantee solvency
- QUESTION: will insurers choose to stay solvent?

- it maximizes expected present value of future revenue
- i.e. chooses at each period $K \in [0, K_{max}]$
  with $K_{max} = \frac{C_{max}}{r} - P$ that

$$\max_K V_0(K) = \int_0^A \left(\frac{V}{r} + K + P - \frac{C}{r} \right) f(C)dC - K$$

(if solvent at $t = 1$: $r(K + P) - C + V$)

- LIMITED LIABILITY $\Rightarrow$ upper limit A
  if $C > A$: insolvent, pays out A, loses $V \Rightarrow$ integrand $= 0$

Optimal reserves (2)

- infinite horizon: future identical at begin. of each period $\Rightarrow V = V_0(K)$ and:

$$V_0(K) = \left[\int_0^A \left(K + P - \frac{C}{r}\right) f(C)dC - K\right] / \left[1 - \frac{F(A)}{r}\right]$$

- put another way: at each period, **IF SOLVENT**, i.e. with proba $F(A)$ gets $\left[\int_0^A \left(K + P - \frac{C}{r}\right) f(C)dC - K\right]$ next period (**DISCOUNTED** at rate $1/r$):

$$V_0(K) = \sum_{t=0}^{+\infty} \left(\frac{F(A)}{r}\right)^t \left[\int_0^A \left(K + P - \frac{C}{r}\right) f(C)dC - K\right]$$

Corner solution

Proposition. If the claims distribution exhibits the increasing failure rate property then the solution of the optimization program of the insurer is a corner solution:

$$K = 0 \quad \text{or} \qquad K = K_{max}$$

Proof: There is no interior maximum:
if $\exists K^* \in (0, K_{\max})/V_0^{'}(K^*) = 0$ then, under the assumption of increasing failure rate, $V_0^{''}(K^*) > 0$

Proof (1): First order condition

$$V_0(K) = \left[\int_0^{r(P+K)} \left(K + P - \frac{C}{r}\right) f(C)dC - K\right] / \left[1 - \frac{F(r(P+K))}{r}\right]$$

$$\Rightarrow V_0^{'}(K) = \frac{1}{\left(1 - \frac{F(A)}{r}\right)^2} \Bigg((-1 + F(A) + 0).\left(1 - \frac{F(A)}{r}\right)$$

$$+ f(A)V_0(K).\left(1 - \frac{F(A)}{r}\right) \Bigg)$$

$$\left(\frac{d}{dx}\int_0^{u(x)} f(x)dx = \int_0^{u(x)} f'(x)dx + f(u(x))u'(x)\right)$$

$$\Rightarrow V_0^{'}(K^*) = \frac{1}{1 - \frac{F(A)}{r}} \left[V_0(K^*)f(A) - (1 - F(A))\right] = 0$$

where $A = r(P + K)$

Proof (2): Second order condition

$$\Rightarrow V_0^{''}(K) = \frac{1}{(1-\frac{F}{r})^2}\left(\left(V_0^{'}(K)f + rV_0(K)f' + rf\right).\left(1 - \frac{F(A)}{r}\right)\right.$$

$$\left. + \left(V_0(K)f - (1-F)\right).f \right)$$

$$= \frac{1}{(1-\frac{F}{r})}\left[V_0^{'}(K)f + rV_0(K)f' + rf + V_0^{'}(K).f\right]$$

$$\Rightarrow V_0^{''}(K^*) = \frac{r}{(1-\frac{F}{r})}\left[V_0(K^*)f' + f\right]$$

▷ what gives using the FOC $V_0^{''}(K^*) = \frac{r}{(1-\frac{F}{r})}\left[\frac{(1-F)}{f}f' + f\right]$

▷ now the ASSUMPTION of increasing failure rate $\frac{d}{dC}\frac{f(C)}{1-F(C)} > 0$ gives $(1-F)f' + f^2 > 0$

▷ $\nexists K^* / V'(K^*) = 0$ and $V^{''}(K^*) \leq 0$

Which corner?

$\Rightarrow$ no interior solution

$\triangleright$ but continuous function on $(0, K_{max}) \Rightarrow \exists$ maximum

$\Rightarrow$ corner solution.

$\triangleright$ Which corner? COMPARE $V_0(0)$ and $V_0(K_{max})$

$$
\begin{aligned}
V_0(0) &= \frac{F(rP)\left(rP - \overline{C_0}\right)}{r - F(rP)} \\
V_0(K_{\max}) &= \frac{rP - \overline{C}}{r - 1} \\
\text{with} \quad \overline{C} &\equiv \mathbb{E}(C) \\
\text{and} \quad \overline{C_0} &\equiv \frac{1}{F(rP)} \int_0^{rP} C dF = \mathbb{E}(C \mid C \leq rP) < \overline{C}
\end{aligned}
$$

Comparison

- ▷ **Advantage** not to put any reserve:
  - ▶ decrease expected claim costs ($\overline{C_0} < \overline{C}$)
  - ▶ due to **limited liability**
- ▷ **Disadvantage**
  - ▶ risk $1 - F(rP) > 0$ of going **out of business**
- ▷ In general, cannot say that a corner **always better**

LIMITATIONS OF THE MODEL

- ▷ interest rate independent of the amount of capital raised
- ▷ no costs associated with raising capital
- ▷ EXOGENEITY OF PREMIUM: willingness to pay for insurance independent of insolvency risk
    - ▶ relaxed in the next model

Failure risk and insurance demand:

see *Rees, Gravelle and Wambach, Regulation of Insurance Markets, GPRIT 1999*

▷ Assume now that policyholders **PERFECTLY OBSERVE** the reserves of their insurer

▷ and can **INFER** from it its failure probability

▷ First: simplest case of **JUST ONE** insurance buyer with income y (earned at end of period $\rightarrow$ "borrow" P)
loss distribution $F(.)$ on $[0, C_u]$
and utility function $u(.)$ with $u^{'} > 0$ and $u^{''} < 0$

$\Rightarrow$ in the absence of insurance: expected utility:

$$\overline{u_0} \equiv \int_0^{C_u} u(y - C)dF$$

Insurance demand

- Assume insurer makes a "take-it-or-leave-it" offer
- "full cover" (repayment=loss) at a premium P
- However, the buyer observes K
- so the premium has to satisfy "participation constraint":

$$\int_0^A u(y - rP)dF + \int_A^{C_u} u(y - C - rP + A)dF \geq \overline{u_0}$$

- note P_0 the **MAXIMAL PREMIUM** the buyer accepts when the insurer has **NO CAPITAL**: $A = rP_0$:

$$P_0 : F(rP_0)u(y - rP_0) + \int_{rP_0}^{C_u} u(y - C)dF = \overline{u_0}$$

and P_u the **MAXIMAL PREMIUM** the buyer accepts when the insurer has **MAXIMUM CAPITAL**: $A = C_u$:

$$P_u : u(y - rP_u) = \overline{u_0}$$

Which corner?

Proposition. When the insurance buyer is fully informed about the insurer's choice of capital; the insurer's expected value is larger at (P_u, K_u) than at $(P_0, K = 0)$.

Proof: we want to show that:

$$\frac{1}{r-1}\int_0^{C_u}(rP_u - C)dF > \frac{1}{r-F(rP_0)}\int_0^{rP_0}(rP_0 - C)dF$$

as $r-1 < r-F$, a sufficient condition would be

$$rP_u - \int_0^{C_u} CdF > F(rP_0)rP_0 - \int_0^{rP_0} CdF$$

$$\text{or}\quad rP_u > F(rP_0)rP_0 + \int_{rP_0}^{C_u} CdF$$

Proof: Jensen inequality (1)

▷ define $\tilde{P} / u(y - r\tilde{P}) = \frac{1}{1-F(rP_0)} \displaystyle\int_{rP_0}^{C_u} u(y - C)dF$

▷ Jensen: $u(.)$ concave $\Rightarrow \forall$ random var. $\tilde{x} : u\left(\mathbb{E}(\tilde{x})\right) > \mathbb{E}(u(\tilde{x}))$

$\Rightarrow r\tilde{P} > \frac{1}{1-F(rP_0)} \displaystyle\int_{rP_0}^{C_u} CdF \Rightarrow (1 - F(rP_0))r\tilde{P} > \int_{rP_0}^{C_u} CdF$

▷ Moreover:

$$\begin{aligned}(1 - F(rP_0))u(y - r\tilde{P}) &= \int_{rP_0}^{C_u} u(y - C)dF \\ &= \overline{u_0} - F(rP_0)u(y - rP_0) \\ &= u(y - P_u) - F(rP_0)u(y - rP_0)\end{aligned}$$

$\Rightarrow u(y - rP_u) = F(rP_0)u(y - rP_0) + (1 - F(rP_0))u(y - r\tilde{P})$

Proof: Jensen inequality (2)

- Using again Jensen's inequality, we have:

$$rP_u > F(rP_0)rP_0 + (1 - F(rP_0))r\tilde{P}$$

- what implies using previous result that:

$$rP_u > F(rP_0)rP_0 + \int_{rP_0}^{C_u} C dF$$

Q.E.D

(a similar result can be proved for any $K < K_u$)

INTUITION

▷ Due to risk aversion ($u(.)$ concave)

▷ policyholder always prepared to pay more than fair premium

▷ to INSURE AGAINST INSURER'S INSOLVENCY

⇒ the insurer (risk-neutral) gains at selling this

⇒ he must put up enough capital to REMAIN SOLVENT

(For now only shown in the simple case of only one buyer)

Generalization to N policyholders

- ▷ need more assumptions
  - ▶ on individual risk
  - ▶ on HOW A IS SHARED in case of failure

- ▷ we assume
  - ▶ i.i.d risk of losing $L(< y)$ with proba p: $C \sim L * \mathcal{B}(n, p)$
  - ▶ in case of failure by the insurer, each policyholder
    - • receive indemnity in full w/ proba A/C
    - • receive noting with proba $(1 - A/C)$

Participation constraint

▷ a policyholder **willing to pay** P for full coverage if:

$$(1-p)u(y-rP)+p\left\{(1-\pi)u(y-rP)\right.$$
$$\left.+\ \pi\Big[(1-\theta)u(y-rP)+\theta u(y-rP-L)\Big]\right\}\geq\overline{u_0}$$

with π: proba insurer insolvent given he suffers the loss

and θ: proba he receives nothing in this case

▷ that is, noting $q\equiv p\pi\theta$

$$(1-q)u(y-rP)+qu(y-rP-L)\geq\overline{u_0}$$

Reserve and failure

▷ Suppose insurer chooses reserves to **meet a given number** $n < N$ of loses. Then:

$$q = p \sum_{m=n-1}^{N-1} \binom{N-1}{m} p^m (1-p)^{N-1-m} \left(1 - \frac{n}{m+1}\right)$$

▷ can then prove equivalent result to previous Proposition

Proposition. If buyers know the probability q that they will not be compensated, the insurer maximizes his expected value by choosing a capital K_m so that there is no default risk ($q = 0$).

$K_m = \frac{N(L - rP_m)}{r}$ w/ P_m largest acceptable premium for $q = 0$

Proof (similar)

- we want to show that, $\forall q > 0$

$$\frac{1}{r-1} N(rP_m - pL) > \frac{1}{r-(1-d)} N(rP_q - (p-q)L)$$

w/ d: default proba; P_q: largest acceptable premium for q

- as $q > 0 \Rightarrow r - 1 < r - (1-d)$, sufficient to show that

$$rP_m \geq rP_q + qL$$

- by definition:

$$u(y - rP_m) = (1-q)u(y - rP_q) + qu(y - rP_q - L) = \overline{u_0}$$

- and Jensen's inequality gives:

$$rP_m > (1-q)rP_q + q(rP_q + L) = rP_q + qL$$

Q.E.D

Intuition (similar)

- policyholders always willing to pay more THAN THE FAIR PREMIUM
- to insure against INSURER'S INSOLVENCY,
- the insurer finds it PROFITABLE to sell him this
- but REQUIRES to put enough capital to remain solvent

CENTRALE MARSEILLE

Conclusions

▷ if policyholders NAIVELY believe that the their insurer would REMAIN SOLVENT

- ▶ might be optimal for insurers NOT TO HOLD RESERVES and to bear FAILURE RISK

▷ BUT if policyholders PERFECTLY INFORMED about insurers failure risk

- ▶ always optimal for insurers to reduce this RISK TO ZERO

⇒ PRINCIPE OF REGULATION: provide policyholders w/ information about insurers failure risk

- ▶ DISCLOSURE on capital, risk exposure,...

+ minimal capital requirement ≈ maximal failure proba

Limitations of the model

- ▷ interest rate independent of the amount of capital raised
- ▷ no costs associated with raising capital
- ▷ impossibility to RECAPITALIZE at the end of each period AFTER claims realization, if $A < C$, insurer might want to raise some capital to REMAIN SOLVENT

Allowing for recapitalization

see *Bourlès and Henriet, 2009*

- ▷ Recall: Why to regulate?
  - ▶ asymmetric information → solution = **DISCLOSURE**
  - ▶ conflict of **INTEREST** betw/ shareholders & policyholders
- ▷ for the insurer to fail:
  - ▶ not only **RESERVES** has to be **INSUFFICIENT**
  - ▶ but also has to be **SUBOPTIMAL** to recapitalize
- ▷ Including shareholders in the model, new choices:
  - ▶ if solvent: take **DIVIDEND** or increase reserves (new shares)
  - ▶ if insolvent: failure or **RECAPITALIZE** (increase reserves)

⇒ information on reserve **NOT SUFFICIENT**

- ▷ failure also depends on recap policy ⇒ **CREDIBILITY ISSUE**

FULL COMMITMENT

- In such a model, the insurance company has to choose
  - how much capital it holds (K)
  - a RECAPITALIZATION POLICY: the interval of claims that will be indemnified (I)
  - an ISSUANCE AND DIVIDEND POLICY
- moreover assume that capital is COSTLY: return on reserves lower than interest rate
- From previous analysis:
  - if insurer can COMMIT EX-ANTE on a recap. policy
  - it commit NEVER TO DEFAULT
  - costly capital $\rightarrow K = 0,\ I = [0, +\infty)$

No commitment

- If insurer cannot **CREDIBLY COMMIT** on I, **EX-POST**:
  - insurer optimally default if amount needed to continue
  - is larger than the present value of the insurance company

- When reserves are **UNOBSERVABLE**, we can show that
  - insurer never holds reserves: $K^* = 0$
  - shareholders take **DIVIDENDS** as soon as possible (never leave money in the insurance company)
  - failure occurs optimally when claims exceed the value of the company

No commitment - observable reserves

- When reserves are **OBSERVABLE**
  - optimal to hold reserves: $K^* > 0$
  - as it increases the maximal acceptable premium
  - failure occurs optimally when claims exceed the value of the company
  - **But**: threshold higher than in previous case: higher premium $\rightarrow$ **HIGHER VALUE**

$\Rightarrow$ **LOWER PROBA OF FAILURE**

Implications for regulation

- ▷ INFORMATION DISCLOSURE gets part of the way
- ▷ RESERVE REQUIREMENT can also be useful:
  by $\uparrow$ the value of the company, it $\downarrow$ the probability of failure
- ▷ But best regulation would be
  - ▶ TO MAKE CREDIBLE the commitment to always recap.
  - ▶ for ex. by setting a GUARANTEE FUND
  - ▶ but... would introduce MORAL HAZARD for shareholders (no incentives to hold reserves)

Insurance regulatory framework: Solvency I

▷ "Current" European regulation: Solvency I

- established in 1973, amended in 2002
- SOLVENCY MARGIN REQUIREMENTS (SMR)
- financial guarantee in addition to provisions
- reserves $>$ SMR $=$ 4% of provisions $+$ 3‰ of capital at risk

▷ Simple and robust framework BUT

- no "true" MEASURE OF RISK taken by the insurer
- no QUALITATIVE requirement (quality of data)
- no DIVERSIFICATION effect
- no role for INFORMATION

Insurance regulatory framework: Solvency II

- New European regulation: Solvency II
  - Reform adopted in 2009 by the European Parliament
  - came into effect on 1 January 2016 (after having been scheduled for 01/01/13 and 01/01/14...)

- Relies as Basel accords on 3 pillars:
  - Pillar I: Quantitative requirements
  - Pillar II: Qualitative requirements
  - Pillar III: disclosure and transparency requirements

Pillar I QUANTITATIVE REQUIREMENT	Pillar II QUALITATIVE REQUIREMENT	Pillar III DISCLOSURE REQUIREMENT
▷ Asset evaluation ▷ Risk definition ▷ Evaluations of ▶ technical provisions ▶ "target" capital (SCR) ▶ minimum capital (MCR)	▷ Internal control ▷ Risk management ▷ Reinforcement and harmonization of external control at EU level	▷ Requirement for standardized information for market authority regulators, investors and policyholders ▷ transparency of financial reporting

Two levels of capital requirement

- SCR (Solvency Capital Requirement)
  - capital required to ensure that insurance company able
  - to **ABSORB SIGNIFICANT UNEXPECTED EVENTS** (bicentennial event)
  - and **GUARANTEE SOLVENCY** in face of such events
  - If capital $<$ SCR: insurance is required to $\uparrow$ capital
  - **TARGETED** value of capital
- MCR (Minimal Capital Requirement)
  - level for which insurer's activity pose an
  - **UNACCEPTABLE RISK** to policyholders
  - If capital $<$ MCR: license withdrawn
    & liabilities transferred to another insurer

How is the SCR calculated?

▷ Risk measure

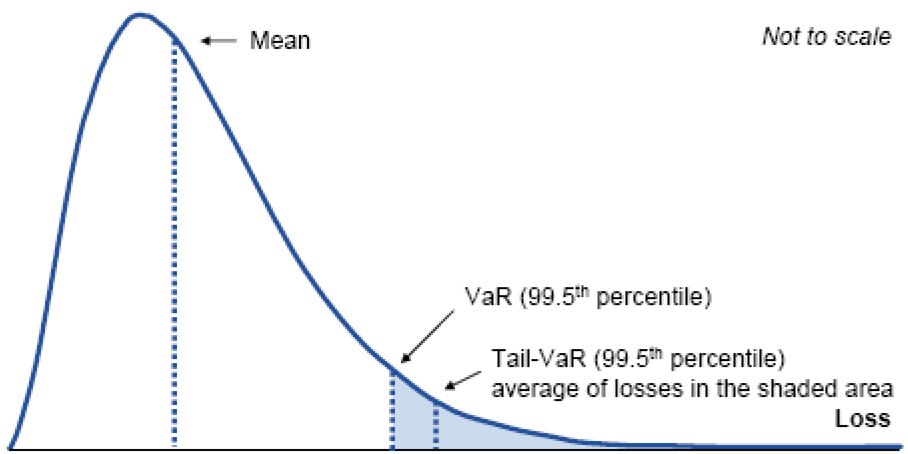
- V@R: value at risk
- Potential loss to be suffered on a portfolio over a given period with a given probability α
- = quantile of loss-and-profit distribution X (asset variation; in our model $X = nP - C$):

$$\mathbb{P}\Big(\, \mathrm{V@R}_{1-\alpha} < X \Big) = \alpha$$

▷ Calibration of the SCR

- SCR = Value-at-Risk at 99.5% over 1-year
- failure probability on 1 year < 0.5%
- able to absorb bicentennial (adverse) event

Value at Risk at 99.5%



Is it a good measure of risk?