# Risk-Taking and Risk-Sharing Incentives under Moral Hazard<sup>†</sup>

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This paper explores the effect of moral hazard on both risk-taking and informal risk-sharing incentives. Two agents invest in their own project, each choosing a level of risk and effort, and share risk through transfers. This can correspond to farmers in developing countries, who share risk and decide individually upon the adoption of a risky technology. The paper mainly shows that the impact of moral hazard on risk crucially depends on the observability of investment risk, whereas the impact on transfers is much more utility dependent. (JEL D81, D82, D86, G22)

In many circumstances, economic theory hardly explains heterogeneous risktaking behaviors. For instance, in rural economies, the adoption of innovations by farmers, like fertilizers or new crops, varies widely across regions. While heterogeneity of individual characteristics partly explains differences in risk-taking behaviors<sup>1</sup>, interaction between agents may also play a role. Here, we consider interactions emerging from risk-sharing arrangement. Indeed, many studies document that households share risk through informal insurance.<sup>2</sup> Risk sharing may contribute to shape risk taking in two respects. First, there are some clear-cut stylized facts attesting that farmers are risk averse, and therefore that they should take more risk when they are insured. Second, a more subtle argument is that risk sharing can embody a moral hazard issue<sup>3</sup>, which may affect individual risk-taking decisions.

This paper explores the effect of moral hazard in effort on both risk-taking and informal risk-sharing incentives. We consider two risk-averse agents. Each agent manages a project. She can affect the return of her project by choosing the technology risk and by exerting an unobservable and costly effort to increase the probability

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<sup>1</sup>See, for instance, Suri (2011).

<sup>3</sup>For empirical evidence on moral hazard, see for instance Lafontaine (1992); for applications related to agricultural context, see Viswanath (2000) for agricultural contract law in Roman Palestine, or Simtowe and Zeller (2007) in the context of microfinance programs in Malawy.

<sup>&</sup>lt;sup>2</sup>See Townsend (1994), Cox and Fafchamps (2008), Fafchamps and Lund (2003), Fafchamps and Gubert (2007), or Angelucci and De Giorgi (2009).

of success of her project. Moreover, agents interact for risk-sharing purposes, by setting up a risk-sharing contract. In the absence of efficient peer monitoring, sharing revenues may generate a moral hazard problem, that is, they may have no incentives to exert effort. When investment risk is exogenous, a traditional mechanism design fostering incentives to exert effort reduces transfers, leaving agents' revenues more exposed to their own effort. However, when risk taking is endogenous, the level of risk itself may be used to restore incentives to exert effort. Direct intuition suggests that, if this is the only lever used, the mechanism designer needs to increase risk in order to increase the agent's exposure to her revenue. However, such an increase in risk generates an increased need for transfers, while reducing transfers mechanically reduces incentives to take risk. Therefore, the joint use of transfers and risk taking as incentive tools induces a priori ambiguous predictions on risk taking and transfers.

The main finding of the paper is that the impact of moral hazard on risk depends on the observability of risk. First, when risk taking is observable, we prove that moral hazard induces an increase in risk taking compared to the case of observable effort, for any strictly increasing and concave utility function; in other terms, with regard to the first-best investment, we obtain overinvestment in the risky technology. Regarding risk-sharing arrangement, whether transfers increase or decrease compared to the first best is utility dependent. Indeed, increased risk taking creates increased needs for transfers (compared to the first best). Thus, two conflicting forces shape the impact of moral hazard on transfers when there is endogenous risk taking. On the one hand, for a given level of risk, to solve the moral hazard issue agents need to reduce transfers. On the other hand, the increase in risk taking creates an increased need for risk sharing. Actually, the presence of moral hazard can either decrease or increase absolute transfer. We present a simple sufficient condition under which transfers are decreased. The condition states that the ratio of the third derivative of utility over the first derivative is decreasing.<sup>4</sup> This condition is met by the standard utility functions lying in the class of the harmonic absolute risk aversion (HARA) utilities. An interesting case is that of constant absolute risk aversion (CARA) preferences, for which the level of wealth transferred between agents is not affected by the presence of moral hazard. This means that risk taking is, in the CARA case, the sole tool used to solve the moral hazard problem.

Second, when private investment is hidden, moral hazard concerns not only effort but also risk taking itself, meaning that agents can deviate jointly in effort and risk from the first-best contract. We then show that both risk and transfers are in general reduced compared to the first-best contract. In other terms, with regard to the firstbest investment, we obtain underinvestment in the risky technology. In this latter context, a third-best contract that avoids individual deviations in joint effort and risk level needs to be designed. If we restrict attention to transfers not exceeding the transfer that would generate equal sharing, which is documented by empirical evidence, we show that the (symmetric) third-best contract<sup>5</sup> is such that both risk taking and transfers are lower than the first-best ones, meaning that observability of investment

<sup>&</sup>lt;sup>4</sup> The coefficient  $\frac{u'''}{u'}$  was examined in Jindapon and Neilson (2007), Modica and Scarsini (2005), and Crainich and Eeckhoudt (2008). To our knowledge, our article is the first to concentrate on the decrease of that coefficient.

<sup>&</sup>lt;sup>5</sup>The study under hidden investment risk focuses on symmetric contracts.

decisively shapes the impact of moral hazard on risk-taking incentives. However, if we allow transfers to exceed the transfer that generates equal sharing, a new candidate for third best appears, satisfying the condition that risk taking is larger than the first best, and the transfer is larger than the level generating equal sharing.

We now briefly discuss the relationship of this paper to the literature. The notion of risk sharing was first developed by Borch, who modelled risk-sharing agreements as a two-person cooperative agreement similar to ours.<sup>6</sup> He stated the mutualization principle (Borch 1962): under complete information, the optimal agreement makes individual wealth only depend on state of nature insofar as the aggregate wealth in that state is concerned.

It is now widely acknowledged that the presence of moral hazard tends to reduce risk sharing, in the sense that it tends to decrease the amount of transfers among agents. Two papers explore the relationship between mutual insurance and moral hazard in context of development economics. Arnott and Stiglitz (1991) ask whether, in the presence of insurance markets, supplemental informal insurance within the family improves welfare. They model family insurance as transfers within pairs of ex ante identical individuals, who choose an effort that increases the probability of success of the technology. Alger and Weibull (2010) incorporate altruism in a similar model of risk sharing with endogenous effort. Our model contributes to that literature by making risk taking endogenous. That is, incomes are endogenously determined not only by individual effort, but also by risk-taking decisions. Moral hazard is also investigated in the principal-agent literature.<sup>7</sup> This literature frequently models a principal who wants to design an incentive-compatible contract. Regarding applications to development economics, the closest literature is perhaps sharecropping (see Cheung 1969 and Stiglitz 1974), which stresses that moral hazard may explain the existence of such contracts (Reid 1976; Eswaran and Kotwal 1985; Ghatak and Pandey 2000). In this case, the tenant exerts some hidden effort, and thus sharecropping has good incentive properties from the viewpoint of the owner, even though such contract makes the owner bear part of the risk. In particular, the literature on double-sided moral hazard (Reid 1977; Eswaran and Kotwal 1985) reconciles the theory with some stylized facts about the nature of sharecropping contracts. In contrast with this principal-agent literature, we focus here on the incentive-compatible agreement that can emerge between two risk-averse agents (that is, we have an agentagent model). Importantly, the principal-agent approach encompasses delegated portfolio management issues (see, for example, Bhattacharya and Pfleiderer 1985), thus differing from the conventional approach insofar as the agent chooses both a risk-taking decision and an effort. However, in this branch of the literature, the effort exerted by the agent reveals information about the return on assets.<sup>8</sup> Here, we focus on efforts enhancing the probability that the risky assets will perform well. The principal-agent literature also addresses group incentives. Analyzing moral hazard in teams or clubs, Holmstrom (1982) and Prescott and Townsend (2006) model

<sup>7</sup> See Windram (2005) for a recent survey. It should be noted that this literature frequently uses the term "risk taking" for the level of risk faced by the principal given the unknown behavior of the agent. This usage differs from that in our paper, where the term denotes the risk associated with some investment made by an economic agent.

<sup>8</sup>See Stracca (2006) for a recent survey.

<sup>&</sup>lt;sup>6</sup>See Fafchamps (2011) for a recent survey on risk sharing.

situations where agents work on a project whose outcome depends on joint efforts. Used to analyze how firms emerge and operate, these models differ from ours in that the efforts of all agents determine the distribution of the aggregate outcome that has to be split among them. In contrast, in our model the effort of each agent determines the distribution of its own outcome, from which she can transfer wealth to others.

Of course, beyond moral hazard, other factors may reduce risk sharing between households. For instance, and relevant in the context of development economies, the lack of enforceability of contracts may reduce the volume of risk-sharing agreements (Townsend 1994).<sup>9</sup> Indeed, without explicit, legally binding, and credibly enforceable contracts, informal agreements have to take into account the possible default of partners, which may induce some limit on the extent of informal risk-sharing agreements (Coate and Ravaillon 1993; Ligon, Thomas, and Worrall 2002; Dubois, Jullien, and Magnac 2008). Our paper assumes perfect commitment, but moral hazard, and focuses on risk-taking incentives under informal risk sharing.

Our work is also related to the literature on the standard portfolio problem. In this widely used model, a single agent allocates wealth between a risk-free project and a risky one. Our main contribution to this literature is the modeling of moral hazard and risk sharing in the standard portfolio problem. We allow agents to enhance the probability of success of the risky project. Our paper is therefore related to Fishburn and Porter (1976) or Hadar and Seo (1990), who study the effect of a shift of distribution of return on attractiveness of investment.<sup>10</sup>

The rest of this paper is structured as follows. Section I introduces the benchmark model in which both efforts and risks are observable. In Section II, we characterize the optimal incentive-compatible sharing rule and the optimal level of risk taking when effort is hidden. Section III examines the case in which investment risk is hidden. Section V concludes. The programs of the first-best and second-best contracts are presented in respectively Appendices I and II. The characterization of the second-best solution (observable investment risk), as presented in Theorem 1 and Proposition 1, are collected in Appendix III, and the characterization of the third-best solution (hidden investment risk), as presented in Theorem 2, is presented in Appendix IV.

## I. A Benchmark Model: Observable Risk and Effort

Two risk-averse agents face independent investment problems. They can choose the level of risk of their investment, what will be referred to as risk taking. To manage her project, each agent exerts an effort which affects the probability of success of her risky project. Moreover, to cope with volatile revenue, agents can share risk through monetary transfers. In this section, we will assume that the investment risk is observable. For instance, the risky project can represent the adoption of an

<sup>&</sup>lt;sup>9</sup>Other factors may explain limited risk sharing. For instance, Bourlès and Henriet (2012) explain limited risk sharing by asymmetric information. They show that the mutuality principle no longer holds when agents have private information on their individual distribution of wealth if heterogeneity is high and risk-aversion is low.

<sup>&</sup>lt;sup>10</sup> A related paper is Belhaj and Deroïan (2011), who explore interdependent risk-taking decisions of agents on a risk-sharing network. They show that an increase in risk sharing does not necessarily lead to an increase in average risk taking. Adding moral hazard and making risk sharing endogenous, we support this conclusion in a two-agent model where we show that moral hazard can increase risk taking and decrease risk sharing.

agricultural innovation, like a new crop or a new fertilizer. Risk-reducing (costly) effort can represent, among many other things, learning about the new plantation, which may increase the chance to obtain a performing harvest, etc. Further, in village economies, the observability of investment risk is likely to occur when farmers involved in an informal risk-sharing arrangement belong to the same village, or when their respective lands are geographically close.

We first model risk-taking behaviors under informal risk-sharing context, and then we incorporate moral hazard.

*Risk Taking in Autarky.*—An agent *i*, endowed with initial wealth  $\omega^i$ , can invest a share  $\alpha^i \in [0, 1]$  of her wealth in a risky project. We interpret  $\alpha^i$  as the level of risk taken by agent *i*. The remaining part is invested in a risk-free project with gross return normalized to 1. The risky project gives a return  $\mu > 1$  with probability *p* (in the case of success) and 0 otherwise.

Agent *i* can exert a costly effort, which increases the probability of success of her risky project. For instance, a farmer can spend time and money improving skills. When agent *i* exerts effort  $e_i$ , the probability of success  $p_i = p(e_i)$ . For simplicity, we consider only two effort levels,  $\underline{e}$  and  $\overline{e}$ , with  $\underline{e} < \overline{e}$ , respectively leading to probabilities of success  $\underline{p}$  and  $\overline{p}$  such that  $\overline{p} - \underline{p} > 0$ . The risky project being profitable for every effort level, we assume that the expected gross return on investment is higher than unity under low effort, that is,  $\underline{p}\mu > 1$ . Low effort is costless, while the cost for providing high effort, C, is positive. To simplify the setting, we rely on a non monetary separable cost of effort unaffected by wealth. Moreover, the cost of effort is independent of the level of risk taking. This fits well, for instance, with the case of informational cost. Farmers' effort to improve skills may not specifically apply to the proportion of the land on which they intend to plant a new crop variety.

Agents are characterized by a von Neumann-Morgenstern Individual utility  $u(\cdot)$ , which is assumed to be continuous, with continuous derivative, strictly increasing and concave in wealth. We will assume throughout the paper that an isolated agent is interested in exerting the high level of effort. Hence, in the absence of risk sharing, a single agent *i* exerts high effort and selects a risk level  $\alpha^i$  to maximize her expected utility (denoted  $EU^i$ ).<sup>11</sup> The isolated agent *i*'s program is therefore written

(1) 
$$\max_{\alpha^{i}} EU^{i}(\alpha^{i}, \bar{e}) = (1 - \bar{p})u(\omega^{i}(1 - \alpha^{i})) + \bar{p}u(\omega^{i}(1 + (\mu - 1)\alpha^{i})).$$

The solution<sup>12</sup> is given by a risk-taking level  $\alpha_0^i$  such that

(2) 
$$\frac{u'(\omega^{i}(1 - \alpha_{0}^{i}))}{u'(\omega^{i}(1 + (\mu - 1)\alpha_{0}^{i}))} = \frac{\overline{p}}{1 - \overline{p}}(\mu - 1).$$

<sup>11</sup>That is, we suppose that  $\max_{\alpha} i(1-\bar{p})u(\omega^i(1-\alpha^i)) + \bar{p}u(\omega^i(1+(\mu-1)\alpha^i)) - C > \max_{\alpha} i(1-\underline{p}) \times u(\omega(1-\alpha^i)) + \underline{p}u(\omega^i(1+(\mu-1)\alpha^i)).$ 

<sup>12</sup>We do not consider the corner solution  $\alpha_0^i = 1$ , which obtains when  $\frac{u'(0)}{u'(\mu\omega)} \le \frac{p}{1-p}(\mu-1)$ . Inada conditions, which impose  $u'(0) = +\infty$ , are sufficient to avoid such a corner solution.

The LHS of equation (2) is increasing in  $\alpha_0$  for all  $\mu > 1$ , and it is equal to 1 when  $\alpha_0 = 0$ . The condition  $\overline{p}\mu > 1$  implies  $\frac{1-\overline{p}}{\overline{p}} < \mu - 1$ , therefore the solution of problem (1) is unique.

Risk Sharing under Observable Efforts and Observable Risks.—Two individuals, say agent *a* and agent *b*, each having a specific project, face the basic investment problem exposed above, and can share risk through monetary transfers. We suppose that agents have the same utility function. We let  $\omega^a$ ,  $\omega^b$  denote the initial wealths of respectively agent *a*, *b*. These wealths may for instance approximate the initial size of farms, or some money left to buy some production technologies. We assume that transfers and risk levels are observable and contractible (section III relaxes the assumption of observable investment risk). Output levels, i.e., the revenues resulting from individual investments, are also observable and contracts are enforceable.<sup>13</sup> Last, individual efforts are also observable.

For simplicity, we assume that risky projects are uncorrelated, thus probabilities of success of both projects are independent.<sup>14</sup> There are four states of nature: in state 1 both agents succeed, in state 2 agent *a* succeeds and agent *b* fails, in state 3 agent *b* succeeds and agent *a* fails, in state 4 both agents fail. The risksharing contract stipulates a monetary transfer between agents in each state of nature. We let the real number  $\tau_j$ , j = 1, 2, 3, 4, denote the transfer from agent *a* to agent *b* in state of nature  $j(\tau_j < 0$  means that agent *b* transfers revenues to agent *a*). Since there is one transfer per state of nature, transfers are not constrained by a specific contractual form (like cross-shareholding, wages, or fixed rent, for example).

We model a benevolent principal that uses an ex ante utilitarian criterion and puts the same weight on the two agents.<sup>15</sup> When  $\omega^a = \omega^b$ , the solution can also be interpreted as the outcome of a Nash bargaining solution with equal outside option and equal bargaining power.

Let  $EU^i$ , i = a, b, denote agent i's expected utility. We have

$$\begin{split} EU^{a}(\alpha^{a}, \tau_{1}, \tau_{2}, \tau_{3}, \tau_{4}, e_{a}, e_{b}) &= p(e_{a})p(e_{b})u(\omega + (\mu - 1)\alpha^{a}\omega - \tau_{1}) \\ &+ p(e_{a})(1 - p(e_{b}))u(\omega + (\mu - 1)\alpha^{a}\omega - \tau_{2}) \\ &+ (1 - p(e_{a}))p(e_{b})u(\omega - \alpha^{a}\omega - \tau_{3}) \\ &+ (1 - p(e_{a}))(1 - p(e_{b}))u(\omega - \alpha^{a}\omega - \tau_{4}) \end{split}$$

<sup>&</sup>lt;sup>13</sup>For a discussion on enforceability, see Coate and Ravaillon (1993); Ligon, Thomas, and Worrall (2002); or Dubois, Jullien, and Magnac (2008).

<sup>&</sup>lt;sup>14</sup>In practice, there may be some correlations if farmers belong to the same village.

<sup>&</sup>lt;sup>15</sup>Assuming that the principal puts different weights on the two agents entails major technical difficulties.

and

$$\begin{split} EU^{b}(\alpha^{b}, \tau_{1}, \tau_{2}, \tau_{3}, \tau_{4}, e_{b}, e_{a}) &= p(e_{b})p(e_{a})u(\omega + (\mu - 1)\alpha^{b}\omega + \tau_{1}) \\ &+ (1 - p(e_{b}))p(e_{a})u(\omega - \alpha^{b}\omega + \tau_{2}) \\ &+ p(e_{b})(1 - p(e_{a}))u(\omega + (\mu - 1)\alpha^{b}\omega + \tau_{3}) \\ &+ (1 - p(e_{b}))(1 - p(e_{a}))u(\omega - \alpha^{b}\omega + \tau_{4}). \end{split}$$

The benevolent principal wants to find a profile of risk levels and transfers  $(\alpha^{a*}, \alpha^{b*}, \tau_1^*, \tau_2^*, \tau_3^*, \tau_4^*)$  maximizing the following program:

(3) 
$$\max_{(\alpha^a, \alpha^b, \tau_1, \tau_2, \tau_3, \tau_4)} EU^a + EU^b.$$

To make the moral hazard problem interesting, we assume that the cost of effort is low enough so that the solution of the problem when both agents exert high effort yields larger expected utility than when both agents exert low effort. Thus, the contract  $(\alpha^{a*}, \alpha^{b*}, \tau_1^*, \tau_2^*, \tau_3^*, \tau_4^*)$  is the first-best contract.

We present the case where agents have the same initial wealth, i.e.,  $\omega^a = \omega^b = \omega$ ; at the end of Section II, we show that the case of heterogenous initial wealths boils down to our benchmark case with an appropriate change of variable. Since agents have the same utility function, it is easily shown that the second best is symmetric, i.e.,  $\alpha^{a*} = \alpha^{b*} = \alpha^*$ ,  $\tau_1^* = \tau_4^* = 0$ ,  $\tau_2^* = -\tau_3^* = \tau^*$  (see Appendix I). For pedagogical purpose, it is convenient to explain the results graphically under symmetric risk levels and transfers. We let  $\alpha$  denote a symmetric risk level (i.e.,  $\alpha = \alpha^a = \alpha^b$ ), and  $\tau$  a transfer in state of nature 2 (thus  $\tau_3 = -\tau$ ), and we let  $EU(\alpha, \tau, \bar{e}, \bar{e})$  denote the (symmetric) individual expected utility.<sup>16</sup> The first-best contract satisfies the following first-order conditions:

(4) 
$$\begin{cases} \tau^* = \frac{\omega\mu}{2}\alpha^* \\ \frac{\bar{p}u'\left(\omega + \frac{\alpha^*\omega(\mu - 2)}{2}\right) + (1 - \bar{p})u'(\omega - \alpha^*\omega)}{\bar{p}u'(\omega + \alpha^*\omega(\mu - 1)) + (1 - \bar{p})u'\left(\omega + \frac{\alpha^*\omega(\mu - 2)}{2}\right)} = \frac{\bar{p}}{1 - \bar{p}}(\mu - 1) \end{cases}$$

In the plan  $(\alpha, \tau)$ , the first-order conditions of the system without moral hazard are represented by function  $\tau_{ES}(\alpha) = \frac{\alpha\mu\omega}{2}$  (with subscript "ES" for equal sharing) and function  $\tau_{ww}(\alpha) \equiv \alpha_{ww}^{-1}(\alpha)$ , where  $\alpha_{ww}(\tau) = \arg \max_{\alpha} EU(\alpha, \tau, \bar{e}, \bar{e})$  (with

<sup>&</sup>lt;sup>16</sup>That is,  $EU(\alpha, \tau, \bar{e}, \bar{e}) = EU^{a}(\alpha, 0, \tau, -\tau, 0, \bar{e}, \bar{e}) = EU^{b}(\alpha, 0, \tau, -\tau, 0, \bar{e}, \bar{e})$ . For the sake of consistency with the rest of the paper, we maintain the double notation regarding efforts.



Figure 1. The First-Order Conditions and the Incentive Constraint in the Plan  $(\alpha, \tau)$ 

subscript "w" for work). We note that the expected utility is increasing along both curves in the plan  $(\alpha, \tau)$  for any  $\alpha < \alpha^*$ .<sup>17</sup> Their sole intersection corresponds to the first-best contract  $M^* = (\alpha^*, \tau^*)$  (see Figure 1). We thus obtain:

PRELIMINARY RESULT 1: The first-best contract is unique and symmetric in risks and transfers. Moreover, transfers satisfy that agents share their revenue equally in each state of nature, and risk sharing enhances risk taking compared to autarky.

Not surprisingly, agents share wealth equally for risk mutualization purposes, i.e., the exclusive motive for transfers is redistribution,<sup>18</sup> and risk sharing fosters risk-taking incentives.

#### II. Moral Hazard under Observable Investment Risk

In this section, we assume that investment risk is observable, but effort level is hidden. The fact that effort is not observable to the contracting partner can be due to the lack of efficient monitoring between farmers, and this creates a potential moral hazard problem. We solve the maximization problem of the benevolent principal, which consists in selecting an incentive-compatible optimum in risk and transfer. To induce effort, the agent needs to be more exposed to her risk, and the standard tool

<sup>17</sup>Indeed, letting  $EU_1$  (resp.  $EU_2$ ) denote the partial derivative of function EU with respect to  $\alpha$  (resp.  $\tau$ ),  $\frac{\partial EU}{\partial \alpha}(\alpha, \tau_{ES}(\alpha)) = EU_1(\alpha, \tau_{ES}(\alpha)) + EU_2(\alpha, \tau_{ES}(\alpha)) \cdot \frac{\partial \tau_{ES}(\alpha)}{\partial \alpha}$ . Basically, for any  $\alpha < \alpha^*$ , we have  $EU_1(\alpha, \tau_{ES}(\alpha)) > 0$ . Further, for any  $\alpha$ , we have  $EU_2(\alpha, \tau_{ES}(\alpha)) = 0$ . Similarly,  $\frac{\partial EU}{\partial \alpha}(\alpha, \tau_{WW}(\alpha)) = EU_1(\alpha, \tau_{WW}(\alpha)) + EU_2(\alpha, \tau_{WW}(\alpha)) \cdot \frac{\partial \tau_{WW}(\alpha)}{\partial \alpha}$ . For any  $\alpha < \alpha^*$ , we have  $EU_2(\alpha, \tau_{WW}(\alpha)) > 0$ . Further, for any  $\alpha$ , we have  $both EU_1(\alpha, \tau_{WW}(\alpha)) = 0$  and  $\frac{\partial \tau_{WW}(\alpha)}{\partial \alpha} > 0$ .

<sup>18</sup> Hence, the successful agent transfers revenue to the other agent. Cox, Galasso, and Jimenez (2006) provide evidence that the average income of donor households exceeds that of recipient households.

consists in reducing transfers. We argue that an alternative tool may be to increase risk taking. However, risk taking and risk sharing are strategic complements, in the sense that increased risk sharing enhances risk taking and vice versa. The impact of moral hazard on risk sharing and risk taking is thus ambiguous: reducing insurance decreases risk taking by complementarity, and therefore reduces the impact of risk taking on incentives to exert effort. Similarly, increasing risk taking calls for insurance, which limits effort incentives. We analyse in this section how agents use these tools jointly to enhance incentives to exert effort.

The game proceeds as follows. First, a contract specifies risk-taking behaviors and transfers. Second, agents take the risk at the level established in the contract and exert an unobservable effort. Third, nature generates revenues (given the levels of investment and efforts). Last, transfers are enforced on the basis of observable realizations. To make the problem interesting, we assume that the first-best contract is not incentive compatible, meaning that  $EU(\alpha^*, \tau^*, \bar{e}, \bar{e}) - C < EU(\alpha^*, \tau^*, \underline{e}, \bar{e})$ . That is, given that investment risk and transfer are observable, defecting upon effort from the first-best optimum is individually beneficial. All other things being equal, this assumption imposes a lower bound on *C*.

We turn to the description of the maximization program of the benevolent principal. It can be shown that, with identical initial wealths, the second-best contract contains symmetric risk levels and transfers (see Appendix II), and we let  $(\alpha^{**}, \tau^{**})$  represent the second-best optimum (i.e.,  $\alpha^{a**} = \alpha^{b**} = \alpha^{**}, \tau_1^{**} = \tau_4^{**} = 0, \tau_2^{**} = -\tau_3^{**} = \tau^{**})$ . Let  $EU(\alpha, \tau, e_i, e_j)$  be the expected utility of agent *i* when she exerts effort  $e_i$  and agent *j* exerts effort  $e_j$ , under symmetric risk level  $\alpha$  and symmetric transfer  $\tau$ . The second-best contract satisfies<sup>19</sup>

$$\begin{array}{ll} (\alpha^{**},\tau^{**}) \ = \ \underset{(\alpha,\tau)}{\operatorname{arg\,max}} EU(\alpha,\ \tau,\ \overline{e},\ \overline{e}) \\ \text{s.t.} \ EU(\alpha,\ \tau,\ \overline{e},\ \overline{e}) \ - \ EU(\alpha,\ \tau,\ \underline{e},\ \overline{e}) \ \geq \ C. \end{array}$$

**Remark:** In the case of symmetric risk levels and transfers, individual incentives to exert high effort decrease with the effort level of the other agent. Indeed, the incentives to free ride on effort are higher when the possibility of receiving a transfer is high. As an agent will receive a transfer only if the other agent succeeds, the probability of getting a transfer increases with the effort of the other agent. Therefore the incentive to shirk is higher when the other agent exerts high effort.

For the sake of clarity, we let  $EU_f$  (resp.  $EU_s$ ) represent the agent's expected payoff in the event of failure (resp. success) of her own project when the other agent exerts effort  $\bar{e}$ :

$$EU_f(\alpha, \tau, \overline{e}) = (1 - \overline{p})u(\omega(1 - \alpha)) + \overline{p}u(\omega(1 - \alpha) + \tau)$$
$$EU_s(\alpha, \tau, \overline{e}) = (1 - \overline{p})u(\omega(1 + \alpha(\mu - 1)) - \tau) + \overline{p}u(\omega(1 + \alpha(\mu - 1))).$$

<sup>19</sup>Of course, the obtained expected utility should be compared to that in which agents exert low effort, choose the optimal risk level and share revenue equally.

With these notations, the second-best contract is given by

(5) 
$$(\alpha^{**}, \tau^{**}) = \underset{(\alpha, \tau)}{\operatorname{arg\,max}} (1 - \overline{p}) E U_f(\alpha, \tau, \overline{e}) + \overline{p} E U_s(\alpha, \tau, \overline{e})$$
  
s.t.  $E U_s(\alpha, \tau, \overline{e}) - E U_f(\alpha, \tau, \overline{e}) \ge \frac{C}{\overline{p} - \underline{p}}.$ 

The benevolent principal wants to maximize individual overall expected payoffs, maintaining a minimal difference  $\frac{C}{\overline{p} - \underline{p}}$  between expected payoffs in the events of success and of failure (to induce high effort).

Our first observation is that, by concavity of the utility function, the optimal incentive-compatible transfer is basically neither negative, nor in excess of that corresponding to equal sharing. That is,  $\tau^{**} \in \left[0, \frac{\omega\mu}{2}\alpha^{**}\right]$ . Second, to figure out how things work, we depict in the plan  $(\alpha, \tau)$  the first-order conditions of the first-best and the second-best contracts as well as the incentive constraint using a specific example (see Figure 1). The function  $\tau_{IC}(\alpha)$  ("IC" for incentive-constraint) represents the binding incentive constraint, i.e., the equation  $EU_s(\alpha, \tau, \bar{e}) - EU_f(\alpha, \tau, \bar{e}) = \frac{C}{\bar{p} - p}$ . We also define its reciprocal function  $\alpha_{IC}(\tau) \equiv \tau_{IC}^{-1}(\tau)$ . The second-best contract is defined as the intersection of  $\tau_{IC}(\alpha)$  and  $\tau_L(\alpha)$  ("L" for Lagrangian), where the function  $\tau_L(\alpha)$  is defined in the proof of Theorem 1. The second-best contract corresponds to the contract  $M^{**} = (\alpha^{**}, \tau^{**})$  in Figure 1. Three remarks are in order. First, functions  $\tau_{ES}(\alpha), \tau_{ww}(\alpha)$ , and  $\tau_{IC}(\alpha)$  are increasing in the plan  $(\alpha, \tau)$ . Second, given the assumption that the first-best contract is not incentive compatible, we have  $\tau_{IC}(\alpha^*) < \tau^*$ . Third, as agents exert high effort under autarky, the risk level  $\alpha_0 = \arg \max_{\alpha} EU(\alpha, 0, \bar{e}, \bar{e})$ , is such that  $\tau_{IC}(\alpha_0) > 0$ .

As a preliminary exploration, we carry out the following local analysis. From the first-best contract let us modify transfer  $\tau$ , keeping risk constant in order to restore incentive compatibility. This corresponds to moving from contract  $M^*$  to contract  $M' = (\alpha^*, \tau_{IC}(\alpha^*))$  in Figure 1. Restoring incentives to effort basically requires lower insurance, thus  $\tau_{IC}(\alpha^*) < \tau^*$ . Now, from contract M', how does expected utility vary locally along the incentive-constraint represented by function  $\tau_{IC}(\alpha)$ ? In particular, does it increase when both transfer and risk increase, or when both decrease? The issue is a priori ambiguous. Indeed, after a simultaneous increase in risk and transfer, both  $EU_f$  and  $EU_s$  are affected in opposite directions. Increasing risk level  $\alpha$  is detrimental to  $EU_f$  and beneficial to  $EU_s$ . Conversely, increasing transfer  $\tau$  is detrimental to  $EU_s$  and beneficial to  $EU_f$ . We find:

LEMMA 1: Consider the contract  $(\alpha^*, \tau_{IC}(\alpha^*))$ . For this contract, expected utility increases locally along the incentive constraint. That is,  $\frac{dEU(\alpha^*, \tau_{IC}(\alpha^*), \bar{e}, \bar{e})}{d\alpha} > 0$ .

Focusing on symmetric contracts, Lemma 1 implies that the expected utility is locally increasing along the incentive constraint around contract M'. The next theorem is much more general. First, it shows that focusing on symmetric contracts

can be made without loss of generality, and second it states that the result given in Lemma 1 is not only local, but also global (proof in Appendix III):

THEOREM 1: The second-best contract is symmetric; that is,  $\alpha^{a**} = \alpha^{b**} = \alpha^{**}$ ,  $\tau_1^{**} = \tau_4^{**} = 0$ ,  $\tau_2^{**} = -\tau_3^{**} = \tau^{**}$ . This contract involves an increase in risk taking compared to the first-best contract, i.e.,  $\alpha^{**} > \alpha^*$ .

The message of the theorem is simple. To increase incentives to exert effort, the contract must increase the dependence of after-transfer revenue on effort. Under exogenous risk taking, the appropriate mechanism is to reduce transfers. However, when risk is endogenous, the appropriate mechanism consists in both increasing risk and reducing relative transfers (compared to the transfer inducing equal sharing). That is, moral hazard leads to overinvestment in risky technology. This theorem provides therefore an explanation to heterogenous risk taking relying on strategic incentives rather than heterogenous characteristics. In our model, agents bear substantial risk to solve moral hazard, using risk taking and risk sharing as joint tools.<sup>20</sup>

**Remark:** We note that the second best can be interpreted as a cross-shareholding contract. That is, it can be achieved if each agent holds a share say  $\beta$  in the other's project, with  $\beta = \frac{\tau}{\alpha\mu\omega}$ .

Although the impact of moral hazard on risk taking entails a systematic enhancement for all strictly increasing and concave utilities, there is no clear-cut impact on absolute transfer. Indeed, while reducing transfers contributes to solving moral hazard, increasing risk also entails an increased need for transfers for the sake of mutualization. Technically, as can be seen in Figure 1, if the function  $\tau_L(\alpha)$  is decreasing (resp. increasing) with the level of risk for all  $\alpha > \alpha^*$ , then  $\tau^{**} < \tau^*$  (resp.  $\tau^{**} > \tau^*$ ). The following proposition gives a condition guaranteeing that the transfer at the second best decreases with regard to the first best (proof in Appendix III):

**PROPOSITION** 1: If the ratio  $\frac{u'''}{u'}$  is decreasing, the second-best contract involves a decrease in (absolute) transfers compared to the first-best contract.

The ratio  $\frac{u'''}{u'}$  can be written as the product  $P \cdot A$ , where  $P = \frac{-u'''}{u''}$  is the index of absolute prudence and A the index of absolute risk aversion. The condition given in Proposition 1 is met by many utility functions. For instance, it holds for quadratic utilities, for constant relative risk aversion (CRRA) utilities and for constant absolute risk aversion (CARA) utilities (for which the ratio is constant). More generally, it is satisfied by most hyperbolic absolute risk aversion (HARA) utilities. HARA utilities are such that  $u(c) = \epsilon \left(\eta + \frac{c}{\gamma}\right)^{1-\gamma}$ ,  $\eta + \frac{c}{\gamma}$ ,  $\epsilon \frac{1-\gamma}{\gamma} > 0$ . This class includes CRRA ( $\eta = 0$  and  $\gamma \ge 0$ ), CARA ( $\gamma \to +\infty$ ) and quadratic ( $\gamma = -1$ )

<sup>&</sup>lt;sup>20</sup>See Hopenhayn and Vereshchagina (2009) for an alternative explanation, based on a dynamical model of occupational choice.



FIGURE 2. THE CARA CASE—TRANSFER IS NOT AN INCENTIVE TOOL

utility functions as special cases. The ratio  $\frac{u''(c)}{u'(c)} = \frac{\gamma+1}{\gamma} \left(\eta + \frac{c}{\gamma}\right)^{-2}$  is decreasing in *c* as soon as  $\gamma \ge -1$ .

A few related remarks follow. First, the condition given in Proposition 1 is necessary for decreasing and convex risk-aversion. Second, if agents are prudent, i.e., u''' > 0, this amounts to having an index of temperance  $\left(\frac{-u'''}{u'''}\right)$  larger than the index of risk-aversion (temperance is used in Eeckhoudt, Gollier, and Schlessinger (1996) and in Gollier and Pratt 1996).<sup>21</sup>

The case of CARA utilities case is eloquent, since the presence of moral hazard does not affect transfers; i.e., only risk taking is used as an incentive tool.

**Example 1 (CARA Utilities):** In the case of CARA utilities, the optimal (absolute) transfer is not impacted by the presence of moral hazard (see proof in the Appendix). For  $u(\omega) = \frac{-1}{\rho} \exp(-\rho\omega)$ , where  $\rho$  is the index of absolute risk aversion,  $\tau^* = \tau^{**} = \frac{1}{\rho} \ln\left[(\mu - 1)\frac{\overline{p}}{1 - \overline{p}}\right]$ . The optimal transfer is thus increasing in level of risk-aversion  $\rho$ , probability of success  $\overline{p}$  and return on the risky technology  $\mu$  (see Figure 2).

Hence, with CARA utilities, solving moral hazard entails no impact on transfers. Indeed, the increase in risk taking aimed at solving moral hazard modifies wealth. This usually modifies the attitude of agents toward risk, and transfers purge this

<sup>&</sup>lt;sup>21</sup> This model echoes the literature on background risks. Eeckhoudt, Gollier, and Schlessinger (1996) look at the effect of a zero-mean background risk (that is, a risk uncorrelated with the return of the risky asset) in the basic standard portfolio problem (without risk sharing). Here, risk sharing can be understood as an additional risk, which corresponds to the risk of having to transfer (or receiving) some wealth. However, this additional risk is clearly negatively correlated with the return on the risky asset: the probability of receiving (resp. making) a transfer is high when the realized return on the risky asset/project is low (resp. high). Eeckhoudt, Gollier, and Schlessinger (1996) show that a zero-mean background risk reduces the demand for the risky asset if absolute risk-aversion is decreasing and convex. Moreover, focusing on unfair background risks, Gollier and Pratt (1996) find that "a necessary and sufficient condition for every small unfair background risk to increase the risk premium is that both absolute prudence and absolute temperance be larger than absolute risk-aversion."

second-order effect, with the exception of the CARA case, for which absolute risk aversion is independent of wealth.

As stated earlier, the inequality  $\tau^{**} < \tau^*$  holds in most cases, but  $\tau^{**} \ge \tau^*$  is also a possible outcome, as illustrated by the following example:

**Example 2:** Consider  $u(\omega) = -\frac{1}{a}\exp(-a \cdot \omega) + k \cdot \omega^4$ , with k = 0.00005, a = 0.1. In this case u is concave for  $\omega < 8$ . We then find  $\tau^{**} > \tau^*$  for  $\omega = 5$ ,  $\mu = 2.2$ ,  $\bar{p} = 0.455$ ,  $\frac{C}{\bar{p} - p} = 1.9$  (the example satisfies our underlying assumptions, and notably the fact that an isolated agent is interested in exerting high effort).

To sum up, under observable investment risk and hidden effort, risk taking increases compared to the first best while transfers may either decrease or increase. Transfers decrease for many utilities, and they are unaffected under CARA utility.

*Heterogeneous Initial Wealths.*—So far, we have analyzed how moral hazard affects investment risk and informal insurance among two homogenous agents with identical initial wealth. However, risk sharing may not necessarily involve households with similar wealths, and it is important to understand how wealth gap affects the results.

Assume  $\omega^a \neq \omega^b$ . Consider both the first-best contract  $(\alpha^{a*}, \alpha^{b*}, \tau_1^*, \tau_2^*, \tau_3^*, \tau_4^*)$ and the second-best contract  $(\alpha^{a**}, \alpha^{b**}, \tau_1^{**}, \tau_2^{**}, \tau_3^{**}, \tau_4^{**})$ . By making the change of variable where, for  $i = 1, 2, 3, 4, \overline{\tau}_i = \tau_i - \frac{\omega^a - \omega^b}{2}$ , both first-best and second-best contract can be obtained from those, noted respectively  $(\overline{\alpha}^*, \overline{\tau}^*)$  and  $(\overline{\alpha}^{**}, \overline{\tau}^{**})$ , of the game with homogeneous initial wealth equal to  $\overline{\omega} = \frac{\omega^a + \omega^b}{2}$ .<sup>22</sup> Regarding risk taking, at both first-best and second-best contracts, the amounts of investment in the risky projects are constant across agents and across games, i.e.,  $\alpha^{a*}\omega^a$  $= \alpha^{b*}\omega^b = \overline{\alpha}^*\overline{\omega}$  and  $\alpha^{a**}\omega^a = \alpha^{b**}\omega^b = \overline{\alpha}^{**}\overline{\omega}$ . Hence, the conclusion of Theorem 1 still holds under heterogeneous initial wealths,  $\alpha^{a*} < \alpha^{a**}$  and  $\alpha^{b*} < \alpha^{b**}$ . Concerning transfers, at the first-best contract, we obtain equal sharing in every state of nature. At the second-best contract, transfers in states 1 and 4 entail equal sharing, while transfers in states 2 and 3 are chosen to create incentives (both are reduced with regard to equal sharing of wealths). We also note that the agent with higher initial wealth gives more to the other if she succeeds and the other fails than the amount she receives when she fails and the other succeeds since  $\tau_2^{**} + \tau_3^{**} = \omega^a - \omega^b$ .

#### III. Moral Hazard under Hidden Investment Risk

In the preceding section, we found that the presence of moral hazard linked to risk-reducing effort enhances risk-taking incentives, irrespective of the shape of utilities. Key to this result is that investment risk is observable, while effort is not. However, in some circumstances, risk taking itself is hidden. For example, distant farmers (e.g., situated in separate villages or regions) may not have detailed

<sup>&</sup>lt;sup>22</sup>An interior solution of the game with heterogeneous initial wealths obtains when the difference of initial wealths is not too large. If the solution is not interior then the poor will invest all her wealth in risky project.

knowledge about the production technologies used by others, like their innovation content.<sup>23</sup> In this case, geographical distance may explain risk unobservability. Another example may concern fishers grouped into cooperatives: in this latter case, each fisher explores her own fishing area, and the technological production by itself can make risk level hidden. When risk is unobservable, agents may see no advantage in conforming with the risk choice of the benevolent principal. Hence, strategic incentives for agents to deviate from the prescribed level of risk should be incorporated. This section examines risk taking and transfers under moral hazard linked to both risk-reducing efforts and risk-taking decisions.

The modified game proceeds as follows. First, a contract specifies transfers; second agents exert a hidden risk-reducing effort and select an unobservable risk level; third nature generates revenues; last, transfers are enforced on the basis of observable realizations. We seek for a contract that guarantees high effort (incentive compatibility). In particular, given that risk levels are hidden, under the third-best contract agents should gain no advantage from deviating jointly in effort and risk level; that is, the actions of each agent should be a Nash equilibrium (i.e., each agent plays a best-response strategy to the current strategy of the other agent).

We restrict attention to symmetric contracts. Let  $\alpha_{sw}(\tau) = \arg \max_{\alpha} EU(\alpha, \tau, \underline{e}, \overline{e})$ ("s" for shirk). This represents the optimal level of risk chosen by an agent when the transfer is fixed to  $\tau$ , she shirks and the other works. Let function  $\tau_{sw}(\alpha) \equiv \alpha_{sw}^{-1}(\alpha)$ . The symmetric optimal contract is then the solution of the program<sup>24</sup>

(6) 
$$(\alpha^{***}, \tau^{***}) = \underset{(\alpha, \tau)}{\operatorname{arg\,max}} EU(\alpha, \tau, \overline{e}, \overline{e})$$
  
s.t.  $EU(\alpha, \tau, \overline{e}, \overline{e}) - EU(\alpha_{sw}(\tau), \tau, \underline{e}, \overline{e}) \ge C$ 

Note that the incentive constraint is more demanding in the program (6) than in the case of observable investment risk program (5). Indeed, the constraint is equivalent to  $EU(\alpha, \tau, \overline{e}, \overline{e}) - C \ge EU(\alpha_i, \tau, \underline{e}, \overline{e})$  for all  $\alpha_i$ , which includes the incentive constraint of the second best ( $\alpha_i = \alpha$ ). Further, we note that the second-best contract ( $\alpha^{**}, \tau^{**}$ ) is not an optimum of this program. Intuitively, agents would be likely to reduce their risk-taking behavior as they are not well enough insured by the second-best transfer.

To start with, we restrict attention to the case in which transfers do not exceed the transfer corresponding to equal sharing. This means that, ex post, the successful agent should be richer than the agent that fails, which is strongly supported by empirical evidence. This restriction therefore appears rather weak. We define the function

$$H(\tau) = EU(\alpha_{ww}(\tau), \tau, \overline{e}, \overline{e}) - C - EU(\alpha_{sw}(\tau), \tau, \underline{e}, \overline{e}).$$

<sup>&</sup>lt;sup>23</sup> A basic incentive to share risks originated from plantations in separate areas is that the likelihood for respective risks to be uncorrelated is higher compared to plantations located in a same area (see Bramoullé and Kranton 2007).

<sup>&</sup>lt;sup>24</sup> As in the preceding section, we assume that the optimal contract obtained when both agents exert low effort generates less utility than the optimum of this program.



Figure 3. The Contract ( $\alpha^{***}, \tau^{***}$ )

Hence, given a transfer  $\tau$ , agents find it profitable to lower both risk taking and effort if and only if  $H(\tau) < 0$ . To get the symmetric third-best contract, we need to define the contract  $(\alpha_c, \tau_c)$ , with  $\tau_c = \max\{\tau < \tau^* \text{ s.t. } \alpha_{ww}(\tau) = \alpha_{IC}(\tau)\}$  and  $\alpha_c = \alpha_{ww}(\tau_c)$ .<sup>25</sup> We obtain the following theorem (proof in Appendix IV):

THEOREM 2: The optimal symmetric contract of the program (6) satisfies

 $(\alpha^{***}, \tau^{***}) = (\alpha_{ww}(\tau^{***}), \max\{\tau < \tau_c \text{ s.t. } H(\tau) = 0\})$ 

When transfers do not exceed the transfer corresponding to equal sharing, the symmetric optimal (third-best) contract is such that both risk taking and transfer are lower than in the first best.

A graphical intuition for why Theorem 2 holds is given in the typical case depicted in Figure 3. First, the contract  $(\alpha^{***}, \tau^{***})$  satisfies that agents are not interested in modifying their risk level, given that both exert high effort, inducing that the contract lies on the curve  $\tau_{ww}(\alpha)$ . Also, the contract must be incentive compatible. This implies that the transfer does not exceed  $\tau_c$ , otherwise agents would exert low effort. Second, and by construction of the second-best incentive constraint, the contract  $(\alpha_c, \tau_c)$  depicted in the figure satisfies that the expected utility when exerting high effort is equal to that when exerting low effort (given that the other agent exerts high effort). From this observation, we can derive that the agent's expected utility for the

<sup>&</sup>lt;sup>25</sup> The transfer  $\tau_c$  exists. The point follows from the observation that  $\tau_{IC}(\alpha_0) > 0$  and that  $\tau_{IC}(\alpha^*) < \tau_{ww}(\alpha^*)$ .

contract  $(\alpha_c, \tau_c)$  when both agents exert high effort is lower than the expected utility for the contract  $(\alpha_{sw}(\tau_c), \tau_c)$  when this agent exerts low effort and the other high effort; this implies that the contract  $(\alpha_c, \tau_c)$  is not a Nash equilibrium. Moreover, we know that an isolated agent is interested in exerting high effort. By continuity, there is at least one risk level  $\tau \in [0, \tau_c[$  such that the agent is indifferent between exerting high effort for the contract  $(\alpha_{ww}(\tau), \tau)$  and low effort for the contract  $(\alpha_{sw}(\tau), \tau)$ . Since the expected utility is increasing along the function  $\alpha_{ww}(\tau)$  for all  $\alpha < \alpha^*$ , we select the contract with the highest risk level, this being the contract  $(\alpha^{***}, \tau^{***})$ .

**Remark:** Allowing transfers to exceed the transfer generating equal sharing, a new candidate for third-best contract can emerge (see proof of Theorem 2 and Figure 5 therein). This new contract satisfies that risk taking exceeds the first-best level, and the transfer exceeds the level of transfer inducing equal sharing, meaning that ex post the agent that fails is richer than the agent that succeeds. Indeed, for any risk level  $\alpha$  larger than the first best, the contract with a transfer generating equal sharing  $\left(\frac{\alpha\mu\omega}{2}, \alpha\right)$  is incentive compatible (risk taking is high enough to enforce effort). Now, that contract should resist to joint deviation in risk and effort. This requires both a high risk level (to foster effort), and a transfer larger than the level corresponding to equal sharing of revenues (to foster the required risk-taking level).

## **IV. Implications**

Our theoretical results have certain implications for the understanding of risk-sharing and risk-taking behaviors in developing economies.

Firstly, our theorems offer an original view for the understanding of some wellknown stylized facts. For instance, many technology adoptions, like the green revolution, are known to be very heterogeneous across space and time (see Evenson and Gollin 2003 and Morris et al. 2007 for related empirical evidences). Our results underline the possible role of investment risk observability. In particular, Theorem 1, when investment risk is observable, may contribute to explain high risk-taking adoption rates, by imputing this finding to the incentive part of risktaking behaviors which may result from the presence of some serious moral hazard issue. Furthermore, there are possible long run implications regarding growth. Indeed, investment choices can affect technologies, and thus impulse some technological changes. It also suggests that, in presence of moral hazard, the risk-taking/ risk-sharing pattern may have an impact on the level of inequalities in the society.

Alternatively, Theorem 2 offers a possible explanation for an empirical puzzle in technology adoption in some developing countries: the low adoption rates of technologies, like hybrid maize or fertilizers, that would strongly increase average farm profits. Indeed, agents may not easily observe the adoption rates of these new technologies (e.g., the quantity of fertilizer or the proportion of land using hybrid maize) by neighbors. Thus, in presence of moral hazard, hidden investment risk may reduce risk-taking incentives of farmers involved in informal risk sharing. Of course, many other explanations exist. For instance, Suri (2011) provides an alternative explanation based on heterogeneous benefits and costs. Another explanation is based on social learning (see Conley and Udry 2001). In particular, Bandiera and Rasul (2006) point out that farmers may tend to free ride on effort to experiment by neighbors, which reduces incentives to adopt the innovation.

The comparison of Theorems 1 and 2 also offers some implication regarding peer monitoring. In principle, monitoring both effort and risk would be equivalent to both effort and investment risk being observable. However, when monitoring is costly, should agents monitor effort or investment risk? One striking conclusion of the model is that it is never optimal to monitor both effort and risk. When the cost of effort monitoring strategy consists in monitoring only effort, and this induces the first-best level of risk taking (even when it is not observable). Indeed, the first-best level of risk taking maximizes individual utility when both agents exert high effort. Otherwise, in cases where monitoring effort is too costly, it is optimal to monitor investment risk if the gap between the second best and the third best is higher than the cost of monitoring risk.

## V. Concluding Remarks

We have considered a model in which two agents make risky investments and set up optimal transfers jointly in the presence of moral hazard. We have shown that both risk taking and transfers are in general used as incentive tools. When risk taking is observable, we found that, for all strictly increasing and strictly concave utilities, the presence of moral hazard enhances risk-taking incentives. Regarding transfers, we provided a sufficient condition, involving the third derivative of utility, under which transfers are decreased. While this condition is met by many utility functions, absolute transfer is not always decreased. In particular, for CARA utilities, moral hazard has no impact on transfers; it only increases risk taking. We have also shown that things are different when private investment is hidden. In this case, (symmetric) third-best contracts satisfy that both risk taking and transfers are lower than the first best (if we forbid transfers exceeding the transfer corresponding to equal sharing of revenues). These findings suggest not only that investment risk should be incorporated in risk-sharing models, but also that the observability of this risk taking is crucial.

The model can be extended in several directions.

*Risk Measures and Risk-Taking Behaviors.*—This work offers some perspectives for empirical research. The primitives of our model are (i) the accurateness of moral hazard issue on effort, (ii) the propensity of households to use informal insurance to bear risk (rather than credit, savings or extra job opportunities), (iii) the perfectness of commitments, (iv) the status of investment risk observability of risk-sharing partners. To delineate the exact contours of risk-sharing and risk-taking behaviors, all of these primitives have to be simultaneously controlled. In particular, some specificities might result from the urbanization context. First, risk taking may be more observable in close knit villages than in urban areas, where neighboring relationships are often more anonymous and perhaps less durable. Second, risk sharing might be less effective in urban areas (although present, see Alvi and Dendir 2008

for a recent evidence that risk-sharing transfers exist between poor urban households in Ethiopia).<sup>26</sup> Third, and partly due to increased anonymity of neighboring links in towns, moral hazard issues may be more pronounced in urban areas.

Overall, to raise some reasonable conclusions, it would be useful to develop more empirical research on risk measures and risk-taking behaviors. Indeed, while there is now a relatively dense empirical literature in development economics identifying informal risk sharing between households, there is only few attempts to study risk taking (see Fafchamps 2010 for an explanation about the specific difficulties faced by econometricians to measure risk as well as its relationship to risk-taking behaviors). A fortiori, assessing the observability of risk taking may be a difficult task. Of course, at the level of generality of the present model, a risky investment may concern various choices, like education levels, agricultural projects or jobs, and the specific nature of each type of risk, as well as the very context in which agents invest, matters crucially.

Investment Risk Observability versus Income Correlations.-This paper has pointed out the role of observability of investment risk for understanding behaviors. However, research should be deepened to encompass all consequences for the design of policy interventions. To illustrate the difficulties of finding a relevant policy, a basic tradeoff for the policymaker stems immediately from our study. Since second best is more desirable than third best, one may infer that a relevant policy intervention could consist in promoting the observability of private investments. For instance, a policymaker may sponsor risk-sharing arrangements between neighboring farmers, and the induced relevant policy recommendation may require some geographical proximity between parties. However, "if network members live too close to one another, they will not be able to insure against area-specific shocks, and monitoring costs may be too high to take advantage of spatial risk diversification." (Cox and Jimenez 1998). And indeed, ensuring a low level of correlations between risky investments often requires to match farmers across different villages or communities. For a policymaker, this raises a specific tradeoff between the observability of investment risks and the correlation of the returns of the projects. In this regard, one relevant information is the fact that incomes are probably less correlated in towns, suggesting unambiguously that, everything equal, potential dilemmas between observability and correlation are less severe in urban areas than in close knit villages. Adequate policy interventions might then possibly use different tools for villages and urban areas. While promoting observability in urban areas, the policymaker may rather sponsor risk sharing among households with uncorrelated incomes in rural villages.

*Endogenous Matching.*—Whereas this paper focused on a benevolent principal view, it would be interesting to study individual incentives to set up contract. This

<sup>&</sup>lt;sup>26</sup>Many reasons can be advocated: opportunities for multiple jobs may reduce the need for interhousehold risk sharing; it may be more easy to differentiate between exogenous risk from risks influenced by behaviors in rural villages; anonymity, which may be more accurate in urban areas, may deter informal risk sharing; urban areas may facilitate the access to formal insurance.

opens the scope for endogenous matching issue.<sup>27</sup> Would symmetric groups be observed at equilibrium, or are heterogeneous pairings sustainable? In our model, at least two mechanisms might generate negative sorting. A first one relies on risk-aversion and insurance motives. Basically, heterogeneous pairings can emerge if the relatively less risk-averse agent insures the more risk-averse agent; in a polar case, a risk neutral agent may prefer to be matched with a risk-averse agent than a risk neutral agent. This might be linked to heterogeneity in wealths. In particular, under DARA utilities, the richer is less risk averse. Another mechanism for negative sorting might be obtained if one agent is more able to bargain a share of the surplus than the other (in the spirit of the mechanism described in Gathak and Karavainov 2011). For instance, under heterogeneous wealths, the richer might be more able to capture the surplus (because, for example, of better outside options).

*Other Applications.*—While the present paper focused on the context of farmers in developing villages, our model is highly stylized and utilities are general. Hence, the basic ingredients of our setting may be found in other economic situations, like over-the-counter (OTC) contracts in financial markets, or the remuneration scheme of portfolio managers in a hedge fund. It would be challenging to explore further such applications.

## APPENDIX I. FIRST-BEST PROGRAM

The following notations are useful: for  $i \in \{a, b\}$ ,  $\omega_0^i = \omega(1 - \alpha^i)$ ,  $\omega_1^i = \omega(1 + (\mu - 1)\alpha^i)$ ,  $\omega_0^{i*} = \omega(1 - \alpha^{i*})$ ,  $\omega_1^{i*} = \omega(1 + (\mu - 1)\alpha^{i*})$ . The objective function of the benevolent principal is written:

$$W(\tau_1, \tau_2, \tau_3, \tau_4, \alpha^a, \alpha^b) = \overline{p}^2 [u(\omega_1^b + \tau_1) + u(\omega_1^a - \tau_1)] + (1 - \overline{p})^2 [u(\omega_0^b + \tau_4) + u(\omega_0^a - \tau_4)] + \overline{p}(1 - \overline{p}) [u(\omega_1^a - \tau_2) + u(\omega_0^b + \tau_2) + u(\omega_0^a - \tau_3) + u(\omega_1^b + \tau_3)].$$

This generates the six following first order conditions, with respect to respectively  $\tau_1$ ,  $\tau_2$ ,  $\tau_3$ ,  $\tau_4$ ,  $\alpha^a$ ,  $\alpha^b$ :

(7) 
$$u'(\omega_1^{b*} + \tau_1^*) - u'(\omega_1^{a*} - \tau_1^*) = 0$$

(8) 
$$u'(\omega_0^{b*} + \tau_4^*) - u'(\omega_0^{a*} - \tau_4^*)] = 0$$

(9) 
$$u'(\omega_0^{b*} + \tau_2^*) - u'(\omega_1^{a*} - \tau_2^*) = 0$$

<sup>27</sup> See Legros and Newman (2002) for theoretical treatment, Ackerberg and Botticini (2002), Serfes (2005), Ghatak and Karaivanov (2011) for applications in the context of sharecropping, and Ghatak (1999, 2000) for applications to micro finance.

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(10) 
$$u'(\omega_1^{b*} + \tau_3^*) - u'(\omega_0^{a*} - \tau_3^*) = 0$$

(11) 
$$(\mu - 1)\overline{p}[\overline{p}u'(\omega_1^{a*} - \tau_1^*) + (1 - \overline{p})u'(\omega_1^{a*} - \tau_2^*)]$$

$$= (1 - \bar{p}) [\bar{p} u'(\omega_0^{a*} - \tau_3^*) + (1 - \bar{p}) u'(\omega_0^{a*} - \tau_4^*)]$$

(12) 
$$(\mu - 1)\overline{p} [\overline{p} u'(\omega_1^{b*} + \tau_1^*) + (1 - \overline{p})u'(\omega_1^{b*} + \tau_3^*)]$$
$$= (1 - \overline{p}) [\overline{p} u'(\omega_0^{b*} + \tau_2^*) + (1 - \overline{p})u'(\omega_0^{b*} + \tau_4^*)].$$

We find

(13) 
$$\frac{u'(\omega_1^{a*} - \tau_1^*)}{u'(\omega_1^{b*} + \tau_1^*)} = \frac{u'(\omega_1^{a*} - \tau_2^*)}{u'(\omega_0^{b*} + \tau_2^*)} = \frac{u'(\omega_0^{a*} - \tau_3^*)}{u'(\omega_1^{b*} + \tau_3^*)} = \frac{u'(\omega_0^{a*} - \tau_4^*)}{u'(\omega_0^{b*} + \tau_4^*)} = 1.$$

Taking the difference between equation (11) and equation (12), it follows that

(14) 
$$\frac{u'(\omega_1^{a*} - \tau_2^*)}{u'(\omega_1^{b*} + \tau_3^*)} = 1.$$

We have, therefore,

(15) 
$$\frac{u'(\omega_1^{a*} - \tau_2^*)}{u'(\omega_1^{b*} + \tau_3^*)} = \frac{u'(\omega_1^{a*} - \tau_2^*)}{u'(\omega_0^{b*} + \tau_2^*)}$$

and

(16) 
$$\frac{u'(\omega_1^{a*} - \tau_2^*)}{u'(\omega_1^{b*} + \tau_3^*)} = \frac{u'(\omega_0^{a*} - \tau_3^*)}{u'(\omega_1^{b*} + \tau_3^*)}$$

We conclude that  $\omega_1^{b*} + \tau_3^* = \omega_0^{b*} + \tau_2^*$  and  $\omega_1^{a*} - \tau_2^* = \omega_0^{a*} - \tau_3^*$ . Thus, we have  $\omega_1^{b*} - \omega_0^{b*} = \omega_1^{a*} - \omega_0^{a*}$ , which entails  $\alpha^{a*} = \alpha^{b*}$ . So risk levels are identical. Moreover, there is a unique optimal vector of transfers. We find  $\tau_1^* = \tau_4^* = 0$ , and  $-\tau_3^* = \tau_2^* = \frac{\mu\omega\alpha^*}{2}$ . Last, we observe that  $\alpha_0 < \alpha^*$ . Indeed, the optimal risk taking is autarky is  $\alpha_0$  such that  $\frac{u'(\omega^i(1 - \alpha_0^i))}{u'(\omega^i(1 + (\mu - 1)\alpha_0^i))} = \frac{\overline{p}}{1 - \overline{p}} (\mu - 1)$ . We define function g such that

$$g(\alpha) = \frac{\overline{p}u'\left(\omega + \frac{\alpha\omega(\mu - 2)}{2}\right) + (1 - \overline{p})u'(\omega - \alpha\omega)}{\overline{p}u'(\omega + \alpha\omega(\mu - 1)) + (1 - \overline{p})u'\left(\omega + \frac{\alpha\omega(\mu - 2)}{2}\right)}$$

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The first-order conditions for risk taking can be written as  $g(\alpha^*) = \frac{\overline{p}}{1-\overline{p}}(\mu-1)$ . By concavity of function *u*, we have

(17) 
$$\frac{u'(\omega - \alpha\omega)}{u'(\omega + \alpha\omega(\mu - 1))} \ge \frac{\bar{p}u'\left(\omega + \frac{\alpha\omega(\mu - 2)}{2}\right) + (1 - \bar{p})u'(\omega - \alpha\omega)}{\bar{p}u'(\omega + \alpha\omega(\mu - 1)) + (1 - \bar{p})u'\left(\omega + \frac{\alpha\omega(\mu - 2)}{2}\right)}.$$

With equation (2) in mind, this gives  $g(\alpha_0) < \frac{\overline{p}}{1-\overline{p}}(\mu-1)$ , that is  $f(\alpha_0) > 0$ . Therefore, as f is decreasing and  $f(\alpha^*) = 0$ , we get  $\alpha^* > \alpha_0$ .

## APPENDIX II. SECOND-BEST PROGRAM (OBSERVABLE INVESTMENT RISK)

The following notations are useful: for  $i \in \{a, b\}$ ,  $\omega_0^i = \omega(1 - \alpha^i)$ ,  $\omega_1^i = \omega(1 + (\mu - 1)\alpha^i)$ ,  $\omega_0^{i**} = \omega(1 - \alpha^{i**})$ ,  $\omega_1^{i**} = \omega(1 + (\mu - 1)\alpha^{i**})$ . The objective function of the benevolent principal is written

$$\begin{aligned} W(\tau_1, \, \tau_2, \, \tau_3, \, \tau_4, \, \alpha^a, \, \alpha^b, \, \bar{e}, \, \bar{e}) &= \bar{p}^{\,2} \left[ u(\omega_1^b \, + \, \tau_1) \, + \, u(\omega_1^a \, - \, \tau_1) \right] \\ &+ \, (1 \, - \, \bar{p})^2 \left[ u(\omega_0^b \, + \, \tau_4) \, + \, u(\omega_0^a \, - \, \tau_4) \right] \\ &+ \, \bar{p}(1 \, - \, \bar{p}) \left[ u(\omega_1^a \, - \, \tau_2) \, + \, u(\omega_0^b \, + \, \tau_2) \right. \\ &+ \, u(\omega_1^b \, + \, \tau_3) \, + \, u(\omega_0^a \, - \, \tau_3) \right]. \end{aligned}$$

The Lagrangian with six instruments is

$$\begin{split} L &= W(\tau_1, \, \tau_2, \, \tau_3, \, \tau_4, \, \alpha^a, \, \alpha^b, \, \bar{e}, \, \bar{e}) \\ &+ \, \lambda_1(EU^a(\alpha^a, \, \tau_1, \, \tau_2, \, \tau_3, \, \tau_4, \, \bar{e}, \, \bar{e}) \, - \, EU^a(\alpha^a, \, \tau_1, \, \tau_2, \, \tau_3, \, \tau_4, \, \underline{e}, \, \bar{e}) \, - \, C) \\ &+ \, \lambda_2(EU^b(\alpha^b, \, \tau_1, \, \tau_2, \, \tau_3, \, \tau_4, \, \bar{e}, \, \bar{e}) \, - \, EU^b(\alpha^b, \, \tau_1, \, \tau_2, \, \tau_3, \, \tau_4, \, \underline{e}, \, \bar{e}) \, - \, C) \end{split}$$

We obtain the following ten conditions:

$$(18) \quad (\mu - 1)(\bar{p} + \lambda_{1})[\bar{p}u'(\omega_{1}^{a**} - \tau_{1}^{**}) + (1 - \bar{p})u'(\omega_{1}^{a**} - \tau_{2}^{**})] \\ = (1 - \bar{p} - \lambda_{1})[\bar{p}u'(\omega_{0}^{a**} - \tau_{3}^{**}) + (1 - \bar{p})u'(\omega_{0}^{a**} - \tau_{4}^{**})] \\ (19) \quad (\mu - 1)(\bar{p} + \lambda_{2})[\bar{p}u'(\omega_{1}^{b**} + \tau_{1}^{**}) + (1 - \bar{p})u'(\omega_{1}^{b**} + \tau_{3}^{**})] \\ = (1 - \bar{p} - \lambda_{2})[\bar{p}u'(\omega_{0}^{b**} + \tau_{2}^{**}) + (1 - \bar{p})u'(\omega_{0}^{b**} + \tau_{4}^{**})] \\ (20) \quad \bar{p}[u'(\omega_{1}^{b**} + \tau_{1}^{**}) - u'(\omega_{1}^{a**} - \tau_{1}^{**})] = \lambda_{1}u'(\omega_{1}^{a**} - \tau_{1}^{**}) - \lambda_{2}u'(\omega_{1}^{b**} + \tau_{1}^{**}) \\ \end{cases}$$

(21) 
$$\overline{p}(1-\overline{p})[u'(\omega_0^{b**}+\tau_2^{**}) - u'(\omega_1^{a**}-\tau_2^{**})]$$
$$= \lambda_1(1-\overline{p})u'(\omega_1^{a**}-\tau_2^{**}) + \lambda_2\overline{p}u'(\omega_0^{b**}+\tau_2^{**})$$
$$(22) \qquad \overline{n}(1-\overline{p})[u'(\omega_1^{a**}-\tau_2^{**}) - u'(\omega_1^{b**}+\tau_2^{**})]$$

(22) 
$$p(1-p)[u'(\omega_0^{a**} - \tau_3^{**}) - u'(\omega_1^{a**} + \tau_3^{**})] = \lambda_1 \bar{p} u'(\omega_0^{a**} - \tau_3^{**}) + \lambda_2 (1-\bar{p}) u'(\omega_1^{b**} + \tau_3^{**})$$

$$(23) \ (1-\bar{p})[u'(\omega_0^{b**}+\tau_4^{**})-u'(\omega_0^{a**}-\tau_4^{**})] = -\lambda_1 u'(\omega_0^{a**}-\tau_4^{**})+\lambda_2 u'(\omega_0^{b**}+\tau_4^{**})$$

$$(24) \lambda_1 \ge 0$$

$$(25) \lambda_2 \ge 0$$

(26) 
$$\lambda_1 \cdot \left[ \bar{p} u(\omega_1^{a**} - \tau_1^{**}) + (1 - \bar{p}) u(\omega_1^{a**} - \tau_2^{**}) - \bar{p} u(\omega_0^{a**} - \tau_3^{**}) - (1 - \bar{p}) u(\omega_0^{a**} - \tau_4^{**}) - C \right] = 0$$

(27) 
$$\lambda_2 \cdot \left[ \bar{p} u(\omega_1^{b**} + \tau_1^{**}) + (1 - \bar{p}) u(\omega_1^{b**} + \tau_3^{**}) - \bar{p} u(\omega_0^{b**} + \tau_2^{**}) - (1 - \bar{p}) u(\omega_0^{b**} + \tau_4^{**}) - C \right] = 0.$$

We find

(28) 
$$\frac{u'(\omega_1^{a**} - \tau_1^{**})}{u'(\omega_1^{b**} + \tau_1^{**})} = \frac{\overline{p} + \lambda_2}{\overline{p} + \lambda_1}.$$

We also get

(29) 
$$\frac{u'(\omega_0^{a**} - \tau_4^{**})}{u'(\omega_0^{a**} + \tau_4^{**})} = \frac{1 - \overline{p} - \lambda_2}{1 - \overline{p} - \lambda_1}.$$

The difference between (18) and (19) is written

$$\begin{aligned} (\mu - 1)\overline{p}[(\overline{p} + \lambda_1)u'(\omega_1^{a**} - \tau_1^{**}) &- (\overline{p} + \lambda_2)u'(\omega_1^{b**} + \tau_1^{**})] \\ &+ (\mu - 1)(1 - \overline{p})[(\overline{p} + \lambda_1)u'(\omega_1^{a**} - \tau_2^{**}) - (\overline{p} + \lambda_2)u'(\omega_1^{b**} + \tau_3^{**})] \\ &= (1 - \overline{p})[(1 - \overline{p} - \lambda_1)u'(\omega_0^{a**} - \tau_4^{**}) - (1 - \overline{p} - \lambda_2)u'(\omega_0^{b**} + \tau_4^{**})] \\ &+ \overline{p}[(1 - \overline{p} - \lambda_1)u'(\omega_0^{a**} - \tau_3^{**}) - (1 - \overline{p} - \lambda_2)u'(\omega_0^{b**} + \tau_2^{**})]. \end{aligned}$$

Now, (28) induces  $(\bar{p} + \lambda_1)u'(\omega_1^{a**} - \tau_1^{**}) = (\bar{p} + \lambda_2)u'(\omega_1^{b**} + \tau_1^{**})$  and (29) induces  $(1 - \bar{p} - \lambda_1)u'(\omega_0^{a**} - \tau_4^{**}) = (1 - \bar{p} - \lambda_2)u'(\omega_0^{b**} + \tau_4^{**})$ . We deduce that

$$(\mu - 1)(1 - \bar{p})[(\bar{p} + \lambda_1)u'(\omega_1^{a**} - \tau_2^{**}) - (\bar{p} + \lambda_2)u'(\omega_1^{b**} + \tau_3^{**})]$$
  
=  $\bar{p}(1 - \bar{p} - \lambda_1)u'(\omega_0^{a**} - \tau_3^{**}) - \bar{p}(1 - \bar{p} - \lambda_2)u'(\omega_0^{b**} + \tau_2^{**}).$ 

Further, (22) implies  $\bar{p}(1-\bar{p}-\lambda_1)u'(\omega_0^{a**}-\tau_3^{**}) = (1-\bar{p})(\bar{p}+\lambda_2)u'(\omega_1^{b**}+\tau_3^{**})$ and (21) implies  $\bar{p}(1-\bar{p}-\lambda_2)u'(\omega_0^{b**}+\tau_2^{**}) = (1-\bar{p})(\bar{p}+\lambda_1)u'(\omega_1^{a**}-\tau_2^{**})$ . We conclude that

(30) 
$$\frac{u'(\omega_1^{a**} - \tau_2^{**})}{u'(\omega_1^{b**} + \tau_3^{**})} = \frac{\overline{p} + \lambda_2}{\overline{p} + \lambda_1}.$$

Moreover, we obtain from (21) and (22) that

(31) 
$$\frac{(\bar{p}+\lambda_1)u'(\omega_1^{a**}-\tau_2^{**})}{(\bar{p}+\lambda_2)u'(\omega_1^{b**}+\tau_3^{**})} = \frac{(1-\bar{p}-\lambda_2)u'(\omega_0^{b**}+\tau_2^{**})}{(1-\bar{p}-\lambda_1)u'(\omega_0^{a**}-\tau_3^{**})}$$

Given that

(32) 
$$\frac{(\overline{p} + \lambda_1)u'(\omega_1^{a**} - \tau_2^{**})}{(\overline{p} + \lambda_2)u'(\omega_1^{b**} + \tau_3^{**})} = 1,$$

we find

(33) 
$$\frac{u'(\omega_0^{b^{**}} + \tau_2)}{u'(\omega_0^{a^{**}} - \tau_3^{**})} = \frac{1 - \bar{p} - \lambda_2}{1 - \bar{p} - \lambda_1}$$

Suppose  $\lambda_2 < \lambda_1$  without loss of generality. Then, equation (32) entails  $\omega_1^{a**} - \tau_2^{**} > \omega_1^{b**} + \tau_3^{**}$ , we also get  $\omega_1^{a**} - \tau_1^{**} > \omega_1^{b**} + \tau_1^{**}$  from equation (28),  $\omega_0^{b**} + \tau_4^{**} > \omega_0^{a**} - \tau_4^{**}$  from equation (29), and  $\omega_0^{a**} - \tau_3^{**} < \omega_0^{b**} + \tau_2^{**}$  from equation (33). This contradicts incentive constraints. Indeed, if both are binding, we have

$$\overline{p} \Big[ \underbrace{u(\omega_1^{a**} - \tau_1^{**}) - u(\omega_1^{b**} + \tau_1^{**})}_{\geq 0} \Big] + (1 - \overline{p}) \Big[ \underbrace{u(\omega_0^{b**} + \tau_4^{**}) - u(\omega_0^{a**} - \tau_4^{**})}_{\geq 0} \Big] = D$$

with

$$D = \bar{p} \Big[ \underbrace{u(\omega_0^{a**} - \tau_3^{**}) - u(\omega_0^{b**} + \tau_2^{**})}_{<0} \Big] + (1 - \bar{p}) \Big[ \underbrace{u(\omega_1^{b**} + \tau_3^{**}) - u(\omega_1^{a**} - \tau_2^{**})}_{<0} \Big],$$

and thus we obtain a contradiction. If only one is binding, then  $\lambda_2 < \lambda_1$  imposes  $IC_2 > IC_1$  that is

$$\overline{p} \begin{bmatrix} u(\omega_1^{b**} + \tau_1^{**}) - u(\omega_1^{a**} - \tau_1^{**}) \end{bmatrix} + (1 - \overline{p}) \begin{bmatrix} u(\omega_0^{a**} - \tau_4^{**}) - u(\omega_0^{b**} + \tau_4^{**}) \end{bmatrix} \\ \underbrace{ \sim 0}_{<0} \\ > (1 - \overline{p}) \begin{bmatrix} u(\omega_1^{a**} - \tau_2^{**}) - u(\omega_1^{b**} + \tau_3^{**}) \end{bmatrix} + \overline{p} \begin{bmatrix} u(\omega_0^{b**} + \tau_2^{**}) - u(\omega_0^{a**} - \tau_3^{**}) \end{bmatrix} \\ \underbrace{ \sim 0}_{>0} \\$$

which entails a contradiction. Thus, we have  $\lambda_2 = \lambda_1$ . This induces  $\omega_1^{a**} - \tau_2^{**} = \omega_1^{b**} + \tau_3^{**}$ ,  $\omega_1^{a**} - \tau_1^{**} = \omega_1^{b**} + \tau_1^{**}$ ,  $\omega_0^{b**} + \tau_4^{**} = \omega_0^{a**} - \tau_4^{**}$ , and  $\omega_0^{b**} + \tau_2^{**} = \omega_0^{a**} - \tau_3^{**}$ . This implies that  $\omega(\mu - 1)(\alpha^{a**} - \alpha^{b**}) = \omega(\alpha^{b**} - \alpha^{a**})$ , and thus  $\alpha^{a**} = \alpha^{b**}$ . We conclude that  $\tau_1^{**} = \tau_4^{**} = 0$  and  $\tau_2^{**} = -\tau_3^{**}$ .

# Appendix III. Characterization of the Second-Best Solution (Observable Investment Risk)

*The Lagrangian.*—Given that the second best is symmetric, we focus on symmetric risk levels and transfers. The Lagrangian associated with the symmetric program (5) is written as

$$\begin{split} L(\alpha,\tau,\bar{e},\bar{e}) &= (1 - \bar{p})EU_f(\alpha,\tau,\bar{e}) + \bar{p}EU_s(\alpha,\tau,\bar{e}) \\ &+ \lambda(EU_s(\alpha,\tau,\bar{e}) - EU_f(\alpha,\tau,\bar{e}) - C), \end{split}$$

with  $\lambda$  the Lagrange multiplier of the incentive constraint. The maximization program entails respectively  $\frac{\partial L}{\partial \tau} = 0$ ,  $\frac{\partial L}{\partial \alpha} = 0$ , and  $\frac{\partial L}{\partial \lambda} = 0$  (since we assume that the first best is not incentive compatible), that is,

$$\begin{cases} \frac{\bar{p}\,u'(\omega-\alpha\omega+\tau)}{(1-\bar{p})u'(\omega+\alpha\omega(\mu-1)-\tau)} = \frac{\bar{p}+\lambda}{(1-\bar{p})-\lambda} \\ \frac{1}{\mu-1}\frac{(1-\bar{p})u'(\omega-\alpha\omega)+\bar{p}\,u'(\omega-\alpha\omega+\tau)}{(1-\bar{p})u'(\omega+\alpha\omega(\mu-1)-\tau)+\bar{p}\,u'(\omega+\alpha\omega(\mu-1))} = \frac{\bar{p}+\lambda}{(1-\bar{p})-\lambda} \\ (1-\bar{p})u(\omega+\alpha\omega(\mu-1)-\tau)+\bar{p}\,u(\omega+\alpha\omega(\mu-1)) \\ -(1-\bar{p})u(\omega-\alpha\omega)-\bar{p}\,u(\omega-\alpha\omega+\tau) = C \end{cases}$$

This gives

$$(34) \begin{cases} \frac{1-\bar{p}}{\bar{p}} \frac{u'(\omega-\alpha\omega)}{u'(\omega-\alpha\omega+\tau)} + 1 = (\mu-1) \left[ \frac{\bar{p}}{(1-\bar{p})} \frac{u'(\omega+\alpha\omega(\mu-1))}{u'(\omega+\alpha\omega(\mu-1)-\tau)} + 1 \right] \\ EU_s(\alpha, \tau, \bar{e}) - EU_f(\alpha, \tau, \bar{e}) = C \end{cases}$$

We introduce the following notations for convenience:

$$\begin{cases} A(\alpha,\tau) = (1-\bar{p})u'(\omega-\alpha\omega) + \bar{p}u'(\omega-\alpha\omega+\tau) \\ B(\alpha,\tau) = (\mu-1)[(1-\bar{p})u'(\omega+(\mu-1)\alpha\omega-\tau) + \bar{p}u'(\omega+(\mu-1)\alpha\omega)] \\ E(\alpha,\tau) = \bar{p}u'(\omega-\alpha\omega+\tau) \\ D(\alpha,\tau) = (1-\bar{p})u'(\omega+(\mu-1)\alpha\omega-\tau) \end{cases}$$

Let  $V(\alpha, \tau) = \frac{A(\alpha, \tau)}{E(\alpha, \tau)} - \frac{B(\alpha, \tau)}{D(\alpha, \tau)}$  and  $IC(\alpha, \tau) = EU_s(\alpha, \tau, \overline{e}) - EU_f(\alpha, \tau, \overline{e}) - C$ . The system (40) can be written as

(35) 
$$\begin{cases} V(\alpha,\tau) = 0\\ IC(\alpha,\tau) = 0 \end{cases}$$

We let functions  $\tau_L(\alpha)$  and  $\tau_{IC}(\alpha)$  represent the functions describing resp. function  $V(\alpha, \tau) = 0$  and  $IC(\alpha, \tau) = 0$  in the plan  $(\alpha, \tau)$ . We observe that  $\tau^* = \tau_L(\alpha^*)$ .

Function  $\tau_L(\alpha)$  Is Increasing.—The equation  $V(\alpha, \tau) = 0$  takes into account the first derivatives of the Lagrangian with respect to  $\tau$  and  $\alpha$ , and can be written as  $\frac{A(\alpha, \tau)}{E(\alpha, \tau)} = \frac{B(\alpha, \tau)}{D(\alpha, \tau)}$ . Note that function  $V(\alpha, \tau)$  is written

$$V(\alpha,\tau) = \frac{1-\overline{p}}{\overline{p}} \frac{u'(\omega-\alpha\omega)}{u'(\omega-\alpha\omega+\tau)} + 1$$

$$- (\mu - 1) \left[ \frac{\overline{p}}{(1 - \overline{p})} \frac{u'(\omega + \alpha \omega(\mu - 1))}{u'(\omega + \alpha \omega(\mu - 1) - \tau)} + 1 \right].$$

A direct application of the implicit function theorem shows that for any strictly increasing and concave utility, function  $\tau_L(\alpha)$  is increasing.

Binding Incentive Constraint  $\tau_{IC}(\alpha)$  Is Increasing.—The incentive constraint defines an implicit relationship between  $\tau$  and  $\alpha$  with

(36) 
$$\frac{\partial \tau_{IC}(\alpha)}{\partial \alpha} = \omega \cdot \frac{A(\alpha, \tau) + B(\alpha, \tau)}{E(\alpha, \tau) + D(\alpha, \tau)}$$

This ratio is positive. Hence, function  $\tau_{IC}(\alpha)$  is increasing.

PROOF OF LEMMA 1: We prove the following slightly more general statement: Lemma: for any risk level  $\alpha$  such that  $\tau_{IC}(\alpha) < \tau_L(\alpha)$ , then  $\frac{dEU(\alpha, \tau_{IC}(\alpha), \bar{e}, \bar{e})}{d\alpha} > 0$ .

From equation (5) we get  $EU(\alpha, \tau, \bar{e}, \bar{e}) = (1 - \bar{p})EU_f(\alpha, \tau, \bar{e}) + \bar{p}EU_s(\alpha, \tau, \bar{e})$ . Further, the binding incentive constraint is given by  $EU_s(\alpha, \tau, \bar{e}) - EU_f(\alpha, \tau, \bar{e}) = C$ . The expected utility on the incentive constraint can then be written as

$$EU(\alpha, \tau_{IC}(\alpha), \bar{e}, \bar{e}) = EU_s(\alpha, \tau_{IC}(\alpha), \bar{e}) + (1 - \bar{p})C.$$

Let  $EU_{s1}$  (resp.  $EU_{s2}$ ) denote the partial derivative of function  $EU_s(\alpha, \tau, \overline{e})$  with respect to  $\alpha$  (resp.  $\tau$ ). Thus,

$$\frac{dEU(\alpha,\tau_{IC}(\alpha),\bar{e},\bar{e})}{d\alpha} = EU_{s1}(\alpha,\tau_{IC}(\alpha),\bar{e}) + EU_{s2}(\alpha,\tau_{IC}(\alpha),\bar{e}) \cdot \frac{\partial\tau_{IC}(\alpha)}{\partial\alpha}.$$

Note that  $EU_{s1}(\alpha, \tau_{IC}(\alpha), \bar{e}) = \omega B(\alpha, \tau)$  and  $EU_{s2}(\alpha, \tau_{IC}(\alpha), \bar{e}) = -D(\alpha, \tau)$ . Given equation (36), we obtain

$$\frac{dEU(\alpha,\tau_{IC}(\alpha),\bar{e},\bar{e})}{d\alpha} = \omega \frac{B(\alpha,\tau)E(\alpha,\tau) - A(\alpha,\tau)D(\alpha,\tau)}{E(\alpha,\tau) + D(\alpha,\tau)}$$

The sign of function  $V(\alpha, \tau)$  is the same sign as AD - BE, which is the opposite of the sign of  $\frac{\partial EU(\alpha, \tau_{IC}(\alpha), \bar{e}, \bar{e})}{\partial \alpha}$ . But function  $V(\alpha, \tau)$  is increasing in  $\tau$ . Then, for any risk level  $\alpha$ , such that  $\tau_{IC}(\alpha) < \tau_L(\alpha)$ , we have  $V(\alpha, \tau_{IC}(\alpha)) < 0$  and thus  $\frac{dEU(\alpha, \tau_{IC}(\alpha), \bar{e}, \bar{e})}{d\alpha} > 0$ . In particular, we have  $\tau_{IC}(\alpha^*) < \tau_L(\alpha^*)$ . Then  $\frac{dEU(\alpha^*, \tau_{IC}(\alpha^*), \bar{e}, \bar{e})}{d\alpha} > 0$ .

#### **PROOF OF THEOREM 1:**

**Step 1:** We show that  $\forall \alpha \geq 0, \tau_L(\alpha) \notin [\min(\tau_{ES}(\alpha), \tau_{ww}(\alpha)), \max(\tau_{ES}(\alpha), and \tau_{ww}(\alpha))]$ . Graphically, this means that the curve  $\tau_L(\alpha)$  does not cross the area "inbetween" curves  $\tau_{ES}(\alpha)$  and  $\tau_{ww}(\alpha)$ .

Basically, if  $\tau_L(\alpha) > \tau_{ES}(\alpha)$ , then  $\frac{E(\alpha,\tau)}{D(\alpha,\tau)} < \frac{\overline{p}}{1-\overline{p}}$ ; conversely, if  $\tau_L(\alpha) < \tau_{ES}(\alpha)$ , then  $\frac{E(\alpha,\tau)}{D(\alpha,\tau)} > \frac{\overline{p}}{1-\overline{p}}$ . Similarly, if  $\tau_L(\alpha) > \tau_{ww}(\alpha)$ ,  $\frac{A(\alpha,\tau)}{B(\alpha,\tau)} < \frac{\overline{p}}{1-\overline{p}}$ ; conversely, if  $\tau_L(\alpha) < \tau_{ww}(\alpha), \ \frac{A(\alpha,\tau)}{B(\alpha,\tau)} > \frac{\overline{p}}{1-\overline{p}}.$  Thus  $\forall \alpha \ge 0$  such that  $\tau_L(\alpha) \in ]\min(\tau_{ES}(\alpha), \tau_{ww}(\alpha)), \max(\tau_{ES}(\alpha), \tau_{ww}(\alpha))[$ , necessarily  $\frac{A(\alpha,\tau)}{B(\alpha,\tau)} \neq \frac{E(\alpha,\tau)}{D(\alpha,\tau)}.$  Then, by construction of function  $\tau_L(\alpha)$ , we have  $\tau \neq \tau_L(\alpha).$ 

**Step 2:** We show that  $\tau_L(0) > 0$ .

Basically,  $V(0,0) = \frac{1-\bar{p}\mu}{\bar{p}(1-\bar{p})}$ . Since  $1 < \bar{p}\mu$ , we get V(0,0) < 0. As  $\frac{\partial V(\alpha, \tau)}{\partial \tau} > 0$ , we are done.

**Step 3:** We show that for all  $\alpha < \alpha^*$ ,  $\tau_{ES}(\alpha) < \tau_L(\alpha)$ . [Graphically, this means that curve  $\tau_L(\alpha)$  is above curve  $\tau_{ES}(\alpha)$  for all  $\alpha < \alpha^*$ ].

From step 2, we get  $\tau_{ES}(0) < \tau_L(0)$ . We then deduce from step 1 that for all  $\alpha < \alpha^*$ ,  $\tau_{ES}(\alpha) < \tau_L(\alpha)$ .

**Step 4:** We show that  $\alpha^* < \alpha^{**}$ .

Recall that

$$\begin{array}{l} (\tau^{**}, \alpha^{**}) \ = \ \underset{(\tau, \alpha)}{\operatorname{arg\,max}} EU(\alpha, \tau, \overline{e}, \overline{e}) \\ \text{s.t.} \quad EU(\alpha, \tau, \overline{e}, \overline{e}) \ - \ EU(\alpha, \tau, \underline{e}, \overline{e}) \ \ge \ C \end{array}$$

This contract basically exists, because the region of incentive-compatible contracts is nonempty. We observe that, since we assume that the first-best contract is not incentive compatible, we have  $\tau_{IC}(\alpha^*) < \tau^*$ . Then we consider two cases:

**Case 1:** For all  $\alpha \in [0, \alpha^*]$ ,  $\tau_{IC}(\alpha) < \tau_{ES}(\alpha)$ . Then curves  $\tau_{IC}(\alpha)$  and  $\tau_L(\alpha)$  never cross for risk levels lower than the first best. That is, for all  $\alpha \in [0, \alpha^*]$ ,  $\tau_{IC}(\alpha) \neq \tau_L(\alpha)$ . Indeed, we know from step 3 that  $\tau_L(\alpha) > \tau_{ES}(\alpha)$ . Then the second best contract satisfies  $\alpha^* < \alpha^{**}$ , and we are done.

**Case 2:** curves  $\tau_{IC}(\alpha)$  and  $\tau_L(\alpha)$  cross for some risk level lower than the first best. That is, there exists  $\alpha \in [0, \alpha^*]$  such that  $\tau_{IC}(\alpha) = \tau_{ES}(\alpha)$ . Consider then  $\alpha_x = \arg \max\{\alpha \in [0, \alpha^*] \text{ s.t. } \tau_{IC}(\alpha) = \tau_{ES}(\alpha)\}$ . Define  $M_x$  as the contract  $(\alpha_x, \tau_{IC}(\alpha_x))$ . Figure 4 illustrates the case. First, the expected utility at contract M is larger than its value in any point  $(\alpha, \tau)$  such that  $\alpha < \alpha_x$ . Indeed, expected utility basically increases when, fixing  $\alpha$ , we go from any feasible transfer to the transfer corresponding to equal sharing; and further, we know from system (4) that the expected utility increases along function  $\tau_{ES}(\alpha)$  up to the first-best contract  $M^*$ , in particular it is increasing up to contract  $M_x$ . Second, by lemma 1 the expected utility increases along function  $\tau_{IC}(\alpha)$  at contract M', we are done.<sup>28</sup>

<sup>&</sup>lt;sup>28</sup>In fact, there is no local optimum for any  $\alpha$  less than  $\alpha^*$ .





*Note:*  $\tau_{IC(\alpha)}$  crosses  $\tau_{ES}(\alpha)$  for some  $\alpha < \alpha^*$ —arrowheads indicate increase of expected utility.

# **PROOF OF PROPOSITION 1:**

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By the implicit function theorem,  $\frac{\partial \tau_L(\alpha)}{\partial \alpha} < 0$  if and only if  $\frac{\partial V(\alpha, \tau)}{\partial \alpha} > 0$ . Direct computation shows that

$$(37) \quad \frac{\partial V(\alpha, \tau)}{\partial \alpha} = \frac{\omega}{\overline{p}(1-\overline{p})[u'(\omega-\alpha\omega+\tau)]^2[u'(\omega+\alpha\omega(\mu-1)-\tau)]^2} \\ \times \left(-(1-\overline{p})^2 u''(\omega-\alpha\omega)u'(\omega-\alpha\omega+\tau)[u'(\omega+\alpha\omega(\mu-1)-\tau)]^2 \\ + (1-\overline{p})^2 u'(\omega-\alpha\omega)u''(\omega-\alpha\omega+\tau)[u'(\omega+\alpha\omega(\mu-1)-\tau)]^2 \\ - (\mu-1)^2 \overline{p}^2 u''(\omega+\alpha\omega(\mu-1)) u'(\omega+\alpha\omega(\mu-1)-\tau) \\ \times [u'(\omega-\alpha\omega+\tau)]^2 + (\mu-1)^2 \overline{p}^2 u'(\omega+\alpha\omega(\mu-1)) \\ \times u''(\omega+\alpha\omega(\mu-1)-\tau)[u'(\omega-\alpha\omega+\tau)]^2 \right).$$

Let us note  $\omega_f = \omega - \alpha \omega$ ,  $\omega_s = \omega + (\mu - 1)\alpha \omega$ . We have, for  $0 < \tau < \frac{\mu}{2} \alpha \omega$ , that  $\omega_f < \omega_f + \tau < \omega_s - \tau < \omega_s$ . Then we have  $\frac{\partial V(\alpha, \tau)}{\partial \alpha} > 0$  (which would give  $\frac{\partial \tau_L(\alpha)}{\partial \alpha} < 0$ ) if and only if

$$(38) \quad (1-\bar{p})^{2} [u'(\omega_{s}-\tau)]^{2} [u''(\omega_{f})u'(\omega_{f}+\tau) - u'(\omega_{f})u''(\omega_{f}+\tau)] + (\mu-1)^{2} \bar{p}^{2} [u'(\omega_{f}+\tau)]^{2} [u''(\omega_{s})u'(\omega_{s}-\tau) - u'(\omega_{s})u''(\omega_{s}-\tau] < 0.$$

Let  $h(c) = u''(c)u'(c + \tau) - u'(c)u''(c + \tau)$ . Then equation (38) writes

(39) 
$$h(\omega_f) < \left(\frac{(\mu-1)\overline{p}}{1-\overline{p}}\right)^2 \left(\frac{u'(\omega_f+\tau)}{u'(\omega_s-\tau)}\right)^2 h(\omega_s-\tau).$$

For every concave utility function,  $u'(\omega_s - \tau) < u'(\omega_f + \tau)$ . Further,  $\overline{p}\mu > 1$ means  $\frac{(\mu - 1)\overline{p}}{1 - \overline{p}} > 1$ . Therefore, a sufficient condition for equation (39) is  $h(\omega_f) < h(\omega_s - \tau)$ . A sufficient condition for that latter condition obtains if function  $h(\cdot)$  is increasing. But  $h'(c) = u'''(c)u'(c + \tau) - u'(c)u'''(c + \tau)$ . Hence, to obtain  $\frac{\partial V(\alpha, \tau)}{\partial \alpha} > 0$  (and therefore  $\tau^{**} < \tau^*$ ), a sufficient condition is that the ratio  $\frac{u'''}{u'}$  is decreasing.

## PROOF OF EXAMPLE 2:

We will show that  $\tau_L(\alpha)$  is flat for CARA utilities. Basically, from equation (37) we derive that  $\frac{\partial \tau_L(\alpha)}{\partial \alpha} = 0 \Leftrightarrow$ 

$$(1 - \overline{p})^{2} [u'(\omega + \alpha \omega(\mu - 1) - \tau)^{2}] [u''(\omega - \alpha \omega)u'(\omega - \alpha \omega + \tau) - u'(\omega - \alpha \omega)u''(\omega - \alpha \omega + \tau)]$$

$$+ (\mu - 1)^2 \overline{p}^2 [u'(\omega - \alpha \omega + \tau)]^2 [u''(\omega + \alpha \omega(\mu - 1))u'(\omega + \alpha \omega(\mu - 1) - \tau)$$

$$-u'(\omega + \alpha\omega(\mu - 1))u''(\omega + \alpha\omega(\mu - 1) - \tau)] = 0.$$

Since  $\frac{u''(\cdot)}{u'(\cdot)}$  is constant across wealth for CARA utilities,  $\frac{\partial \tau_L(\alpha)}{\partial \alpha} = 0$  for all  $\alpha$ ,  $\tau$ , and  $\tau^* = \tau^{**}$  can be inferred by  $V(0, \tau) = 0$ . The result follows.

# Appendix IV. Characterization of the (Symmetric) Third-Best Solution (Hidden Investment Risk)

#### **PROOF OF THEOREM 2:**

By construction of program (6), the third-best contract  $(\tau^{***}, \alpha^{***})$  is such that  $\tau^{***} = \tau_{ww}(\alpha^{***})$ , and moreover it satisfies that  $\tau^{***} \leq \tau_{IC}(\alpha^{***})$ .

We claim that the third-best contract  $(\tau^{***}, \alpha^{***})$  exists. Basically,  $H(\tau_c) < 0$  and H(0) > 0. By continuity, there exists  $\tau \leq \tau_c$  such that  $H(\tau) = 0$ . Two cases can arise:

**Case 1:** For all intersections between curves  $\tau_{IC}(\alpha)$  and  $\tau_{ww}(\alpha)$ , the risk level is lower that the first best. That is, for all contracts  $(\tau, \alpha_{ww}(\tau))$  such that  $\tau_{ww}(\alpha) = \tau_{IC}(\alpha)$ , we have  $\alpha_{ww}(\tau) < \alpha^*$ .

Then clearly  $\alpha^{***} < \alpha^*$ . Now, if  $\tau > \tau_c$ , the contract  $(\tau, \alpha_{ww}(\tau))$  is not incentive compatible because  $\alpha_{ww}(\tau) < \alpha_{IC}(\tau)$ . If  $\tau \in ]\tau^{***}$ ,  $\tau_c[$ , agents have incentives to reduce both effort and risk level because  $H(\tau) < 0$ . Last, if  $\tau < \tau^{***}$ ,  $EU(\alpha_{ww}(\tau), \tau, \bar{e}, \bar{e})$  is increasing in  $\alpha$ , which means that the contract  $(\tau, \alpha_{ww}(\tau))$  is not an optimum. In the end, the third-best contract is the contract  $(\tau^{***}, \alpha^{***})$  and we are done.

**Case 2:** There exists an intersection of curves  $\tau_{IC}(\alpha)$  and  $\tau_{ww}(\alpha)$  such that the risk level is larger than the first best. That is, there exists a contract  $(\tau, \alpha_{ww}(\tau))$ , with  $\alpha_{ww}(\tau) > \alpha^*$ , such that  $\tau_{ww}(\alpha) = \tau_{IC}(\alpha)$ .

Note that, for all  $\alpha > \alpha^*$ , the contract  $(\tau, \alpha_{ww}(\tau))$  satisfies that  $\tau_{ww}(\alpha) > \tau_{ES}(\alpha)$ , that is the contracts on curve  $\tau_{ww}(\alpha)$  contain transfers larger than the transfer corresponding to equal sharing. Figure 5 depicts a typical Case 2.<sup>29</sup> We now define the transfer  $\tau'_c = \min\{\tau > \tau^* \text{ s.t. } \alpha_{ww}(\tau) = \alpha_{IC}(\tau)\}$ . Then, using a similar argument to that used to establish that  $H(\tau_c) < 0$ , we get  $H(\tau'_c) < 0$ , meaning that the contract  $(\tau'_c, \alpha_{ww}(\tau'_c))$  is not robust to individual deviations in joint effort and risk. Consider now the following two contracts  $(\tau_R, \alpha_R)$  and  $(\tau'_R, \alpha'_R)$  defined as follows:

$$\begin{cases} \tau_R = \max\{\tau \le \tau_c \quad \text{s.t.} \quad H(\tau) = 0\} \\ \alpha_R = \alpha_{ww}(\tau_R) \\ \tau'_R = \min\{\tau \ge \tau'_c \quad \text{s.t.} \quad H(\tau) = 0\} \\ \alpha'_R = \alpha_{ww}(\tau'_R) \end{cases}$$

The contract  $(\tau_R, \alpha_R)$ , which is the equivalent of the third-best contract in Case 1, exists. The contract  $(\tau'_R, \alpha'_R)$  satisfies that  $\tau'_R > \frac{\alpha'_R \mu \omega}{2}$ . This contract may not exist, since it may be that, for all  $\tau \in [\tau'_c, \omega]$ ,  $H(\tau) < 0$ . In that latter situation, the third-best contract is determined as in Case 1, that is, the third-best contract is the contract  $(\tau_R, \alpha_R)$ . If, however, the contract  $(\tau'_R, \alpha'_R)$  exists, the third-best contract is determined as follows.

As with Case 1, the contract  $(\tau_R, \alpha_R)$  is a candidate. If  $\tau < \tau_R$ ,  $EU(\alpha_{ww}(\tau), \tau, \bar{e}, \bar{e})$  is increasing in  $\alpha$ , which means that the contract  $(\tau, \alpha_{ww}(\tau))$  is not an optimum. If  $\tau \in ]\tau_R$ ,  $\tau_c[$ , agents have incentives to lower both effort and risk level because  $H(\tau) < 0$ . If  $\tau \in [\tau_c, \tau^*]$ , the contract  $(\tau, \alpha_{ww}(\tau))$  is not incentive compatible because  $\alpha_{ww}(\tau) < \alpha_{IC}(\tau)$ .

<sup>29</sup>To illustrate, this situation can arise with CARA utility (for instance, set C = 0.0006,  $\omega = 4$ ,  $\mu = 2.5$ ,  $a = 2, \bar{p} = 0.9$ ,  $\underline{p} = 0.83$ ).



Figure 5. The Two Candidates for the Contract ( $\alpha^{***}, \tau^{***}$ )

The contract  $(\tau'_{R}, \alpha'_{R})$  is also a candidate. If  $\tau \in ]\tau^{*}, \tau'_{c}]$ , then  $\alpha_{IC}(\tau) > \alpha_{ww}(\tau)$ , implying that the contract  $(\tau, \alpha_{ww}(\tau))$  is not incentive compatible. If  $\tau \in ]\tau'_{c}, \tau'_{R}[$ , then  $H(\tau) < 0$ , and thus agents have incentives to reduce both effort and risk level. Last, if  $\tau > \tau'_{R}$ ,  $EU(\alpha_{ww}(\tau), \tau, \bar{e}, \bar{e})$  is decreasing in  $\alpha$ , which means that the contract  $(\tau, \alpha_{ww}(\tau))$  is not an optimum. In the end,

$$( au^{***}, \alpha^{***}) = \operatorname*{arg\,max}_{( au_R, lpha_R), ( au'_R, lpha'_R)} EU(lpha, au, \overline{e}, \overline{e}).$$

Now, if we restrict attention to transfers that do not exceed the transfer corresponding to equal sharing, the contract  $(\tau'_R, \alpha'_R)$  should not be taken into account, and therefore  $(\tau^{***}, \alpha^{***}) = (\tau_R, \alpha_R)$ .

#### REFERENCES

- Alger, Ingela, and Jörgen W. Weibull. 2010. "Kinship, Incentives, and Evolution." American Economic Review 100 (4): 1725–58.
- Angelucci, Manuella, and Giacomo De Giorgi. 2009. "Indirect Effects of an Aid Program: How Do Cash Transfers Affect Ineligibles Consumption?" *American Economic Review* 99 (1): 486–508.
- Arnott, Richard J., and Joseph E. Stiglitz. 1991. "Moral Hazard and Nonmarket Institutions: Dysfunctional Crowding Out or Peer Monitoring?" *American Economic Review* 81 (1): 179–90.
- Bandiera, Oriana, and Imran Rasul. 2006. "Social Networks and Technology Adoption in Northern Mozambique." *Economic Journal* 116 (514): 869–902.
- Belhaj, Mohamed, and Frédéric Deroïan. 2012. "Risk taking under heterogenous revenue sharing." Journal of Development Economics 98 (2): 192–202.

Ackerberg, Daniel A., and Maristella Botticini. 2002. "Endogenous Matching and the Empirical Determinants of Contract Form." *Journal of Political Economy* 110 (3): 564–91.

- Bhattacharya, Sudipto, and Paul Pfleiderer. 1985. "Delegated portfolio management." Journal of Economic Theory 36 (1): 1–25.
- Borch, Karl. 1962. "Equilibrium in a Reinsurance Market." Econometrica 30 (3): 424-44.
- Bourlès, Renaud, and Dominique Henriet. 2012. "Risk-sharing Contracts with Asymmetric Information." Geneva Risk and Insurance Review 37 (1): 27–56.
- Bramoullé, Yann, and Rachel Kranton. 2007. "Risk Sharing across Communities." American Economic Review 97 (2): 70–74.
- Cheung, Steven N. S. 1969. The Theory of Share Tenancy. Chicago: University of Chicago Press.
- Coate, Stephen, and Martin Ravallion. 1993. "Reciprocity without commitment: Characterization and performance of informal insurance arrangements." *Journal of Development Economics* 40 (1): 1–24.
- **Conley, Timothy G., and Christopher R. Udry.** 2010. "Learning about a New Technology: Pineapple in Ghana." *American Economic Review* 100 (1): 35–69.
- Cox, Donald, and Marcel Fafchamps. 2008. "Extended Family and Kinship Networks: Economic Insights and Evolutionary Directions." In *Handbook of Development Economics*, Vol. 4, edited by T. Paul Schultz and John Strauss, 3711–86. Amsterdam: North-Holland.
- Cox, Donald, Emanuela Galasso, and Emmanuel Jimenez. 2006. "Private Transfers in a Cross Section of Developing Countries." Boston College Center for Retirement Research (CRR) Working Paper 2006-1.
- **Cox, Donald, and Emmanuel Jimenez.** 1998. "Risk Sharing and Private Transfers: What about Urban Households?" *Economic Development and Cultural Change* 46 (3): 621–37.
- Crainich, David, and Louis Eeckhoudt. 2008. "On the intensity of downside risk aversion." *Journal of Risk and Uncertainty* 36 (3): 267–76.
- **Dubois, Pierre, Bruno Jullien, and Thierry Magnac.** 2008. "Formal and Informal Risk Sharing in LDCs: Theory and Empirical Evidence." *Econometrica* 76 (4): 679–725.
- **Eeckhoudt, Louis, Christian Gollier, and Harris Schlessinger.** 1996. "Changes in Background Risk and Risk Taking Behavior." *Econometrica* 64 (3): 683–89.
- Eswaran, Mukesh, and Ashok Kotwal. 1985. "A Theory of Contractual Structure in Agriculture." *American Economic Review* 75 (3): 352–67.
- Evenson, Robert E., and Douglas Gollin. 2003. "Assessing the Impact of the Green Revolution, 1960 to 2000." *Science* 300 (5620): 758–62.
- Fafchamps, Marcel. 2010. "Vulnerability, risk management, and agricultural development." African Journal of Agricultural Economics 5 (1): 243–60.
- Fafchamps, Marcel. 2011. "Risk Sharing Between Households." In *Handbook of Social Economics*, Vol. 1A, edited by Jess Benhabib, Alberto Bisin, and Matthew O. Jackson, 1255–80. Amsterdam: North-Holland.
- Fafchamps, Marcel, and Flore Gubert. 2007. "The formation of risk sharing networks." Journal of Development Economics 83 (2): 326–50.
- Fafchamps, Marcel, and Susan Lund. 2003. "Risk-sharing networks in rural Philippines." Journal of Development Economics 71 (2): 261–87.
- Fishburn, Peter C., and R. Burr Porter. 1976. "Optimal portfolios with one safe and one risky asset: effects of changes in the rate of return and risk." *Management Science* 22 (10): 1064–73.
- **Ghatak, Maitreesh.** 1999. "Group lending, local information and peer selection." *Journal of Development Economics* 60 (1): 27–50.
- Ghatak, Maitreesh. 2000. "Screening by the Company You Keep: Joint Liability Lending and the Peer Selection Effect." *Economic Journal* 110 (465): 601–31.
- Ghatak, Maitreesh, and Alexander Karavainov. Forthcoming. "Contractual Structure in Agriculture with Endogenous Matching." *Journal of Development Economics*.
- Gollier, Christian, and John W. Pratt. 1996. "Risk Vulnerability and the Tempering Effect of Background Risk." *Econometrica* 64 (5): 1109–23.
- Hadar, Josef, and Tae Kun Seo. 1990. "The Effects of Shifts in a Return Distribution on Optimal Portfolios." *International Economic Review* 31 (3): 721–36.
- Holmström, Bengt. 1982. "Moral Hazard in Teams." Bell Journal of Economics 13 (2): 324-40.
- **Jindapon, Paan, and William S. Neilson.** 2007. "Higher-order generalizations of Arrow–Pratt and Ross risk aversion: A comparative statics approach." *Journal of Economic Theory* 136 (1): 719–28.
- Lafontaine, Francine. 1992. "Agency theory and franchising: some empirical results." *Rand Journal of Economics* 23 (2): 263–83.
- Legros, Patrick, and Andrew F. Newman. 2002. "Monotone Matching in Perfect and Imperfect Worlds." *Review of Economic Studies* 69 (4): 925–42.
- Ligon, Ethan, Jonathan P. Thomas, and Tim Worrall. 2001. "Informal Insurance Arrangements in Limited Commitment: Theory and Evidence from Village Economies." *Review of Economic Studies* 69 (1): 209–44.

- Modica, Salvatore, and Marco Scarsini. 2005. "A note on comparative downside risk aversion." *Journal of Economic Theory* 122 (2): 267–71.
- Morris, Michael, Valerie A. Kelly, Ron J. Kopicki, and Derek Byerlee. 2007. Fertilizer Use in African Agriculture, Lessons Learned and Good Practice Guidelines. Washington, DC: World Bank.
- **Prescot, Edward S., and Robert M. Townsend.** 2006. "Theory of the Firm: Applied Mechanism Design." Federal Reserve Bank of Richmond Working Paper 96-2.
- Reid, Joseph D., Jr. 1976. "Sharecropping and Agricultural Uncertainty." *Economic Development and Cultural Change* 24 (3): 549–76.
- Reid, Joseph D., Jr. 1977. "The Theory of Share Tenancy Revisited—Again." Journal of Political Economy 85 (2): 403–07.
- Serfes, Konstantinos. 2005. "Risk sharing vs. incentives: Contract design under two-sided heterogeneity." *Economics Letters* 88 (3): 343–49.
- Simtowe, Franklin, and Manfred Zeller. 2007. "Determinants of Moral Hazard in Microfinance: Empirical Evidence from Joint Liability Lending Programs in Malawi." Munich Personal RePEc Archive (MPRA) Paper 461.
- Suri, Tavneet. 2011. "Selection and Comparative Advantage in Technology Adoption." *Econometrica* 79 (1): 159–209.
- Stiglitz, Joseph E. 1974. "Incentives and Risk Sharing in Sharecropping." Review of Economic Studies 41 (2): 219–55.
- Stracca, Livio. 2006. "Delegated Portfolio Management: A Survey of the Theoretical Literature" Journal of Economic Surveys 20 (5): 823–48.
- Townsend, Robert M. 1994. "Risk and Insurance in Village India." Econometrica 62 (3): 539-91.
- Vereshchagina, Galina, and Hugo A. Hopenhayn. 2009. "Risk Taking by Entrepreneurs." American Economic Review 99 (5): 1808–30.
- Viswanath, P. V. 2000. "Risk sharing, diversification and moral hazard in Roman Palestine: evidence from agricultural contract law." *International Review of Law and Economics* 20 (3): 353–69.
- Windram, Richard. 2005. "Risk-Taking Incentives: A Review of the Literature." *Journal of Economic Surveys* 19 (1): 65–90.